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Analysis of Magnetized Chemical Reaction under Arrhenius Control in the Presence of Navier Slip and Convective Boundary Conditions

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ABSTRACT
The presence of Navier slip and Newtonian heating conditions at the boundary layer flow affects the shear stress distribution and overall flow behavior which causes the thermal boundary layer to be thinner or thicker depending on the relative strength of the slip velocity and Newtonian heating rate. In this paper, the steady state magnetized chemical reaction under Arrhenius control in the presence of Navier slip and Newtonian heating at both walls is studied. Using appropriate dimensionless quantities and parameters, the governing ordinary differential equations in dimensional form have been reduced to dimensionless governing equations. A semi-analytical solution for velocity and temperature are obtained using semi-analytical approach (regular perturbation method). Expressions of skin friction and rate of heat transfer are also displayed. The results show that Navier slip and Newtonian heating conditions affect the flow pattern significantly.

Keywords:
Magnetohydrodynamics (MHD), Biot number, Arrhenius kinetics, Navier slip effect, Convective boundary condition, Semi-analytical method.

INTRODUCTION

Magneto-hydrodynamic (MHD) fluid are employed in various industrial processes to regulate the speed of cooling and heating, the heat transfer behaviors of nuclear reactors, particularly those using liquid metal cooling systems and also influence astrophysics, propulsion power generation, space exploration and environmental engineering ((2015) and (2020)). Further, magnetic technologies are increasingly appealing in industries because they offer diverse applications in biomaterials for wound treatment, gastrointestinal medications, sterilization devices and related fields. Magnetohydrodynamics (MHD) is a significant field in contemporary scientific research and engineering, focusing on the interaction between magnetic fields imposed from outside and the flow of electrically conducting fluids. It can be viewed as a specialized branch of fluid mechanics (2021). To this end, very recently, Ojemeri et al. (2024) published a study involving the analysis of a mechanized chemically reactive fluid experiencing heat generation or absorption, flowing through a superhydrophobic microchannel that undergoes alternating heating. In a separate work, Ojemeri et al. (2023) explored that analytical solutions are developed for natural convection of a chemically reactive fluid, considering Soret effects and a radial magnetic field within a permeable annular region bounded by concentric cylinders. It is important to highlight that increasing Frank-Kamenetskii, sustentation, and thermo-diffusion parameter enhances fluids velocity whereas decreasing the influence of radial magnetic field diminishes fluid flow. The perturbation series technique was utilized by Taid and Ahmed (2022) To determine the combined effects of Soret and heat dissipation on a chemically driven natural hydrodynamic flow along an inclined permeable channel filled with porous material. Osman et al. (2022) utilized the Laplace transformation method to analyze the behavior of hydrodynamic flow in natural convection through an infinitely inclined channel. Siva et al. (2021) reported on a heat transfer issue involving the electro kinetic effect in an oscillating micro-channel plate, and demonstrated significant variation due to MHD involvement. Sandeep and Sugunamma (2013) studied the effect of an inclined magnetic field on the transient natural convection of a reactive dusty fluid between two infinite plate filled with porous material. Yale et al. (2023) Examined the effects of super-hydrophobicity and MHD on a viscous dissipative fluid in a slit microchannel with a super-hydrophobic coating, employing a regular perturbation method. References such as Joseph et al. ((2015), (2016) and (2023)) further exemplified this phenomenon.

Chemical reaction plays a vital role in MHD, energy transfer and diagnostic plasma behavior in astrophysical phenomena and interacting with magnetic fields, many researchers studies the MHD chemical reaction with difference boundary condition. Considering this, Hamza et al. (2023) Explored the numerical computational treatment of a transient electrically conducting fluid undergoing an Arrhenius reaction, considering Navier slip and Newtonian heating, using an implicit finite difference scheme. They demonstrated that a slight increase in the Hartmann number enhances the Lorentz force which streamline the velocity boundary layer and decelerates the flow. Fenuga et al. (2018) reported the influence of mixed convection and the Navier slip parameter on chemically reactive heat and mass transfer in MHD fluid flow over a permeable surface with convective boundary conditions. Hamza et al. (2021) analyzed the impact of MHD on free convective slip flow of an exothermic fluid in a convectively heated vertical channel, using perturbation method. The author's noted that both fluid velocity and skin friction decreases as the Hartmann number increases. Also, the effect of non-linear thermo-convective Arrhenius Casson fluid flow between two vertical parallel plates influenced by an exothermic chemical reaction as well as linear and non-linear volumetric thermal and concentration expansion was analyzed using the parametric continuous method was demonstrated by Hussain et al. (2022). Ojemeri and Hamza (2022) studied the effect of a chemically reacting fluid in a vertical microchannel under the influence of a transverse magnetic field and heat generation using the homotopy perturbation method. The author discovered that increasing the chemical reaction and rarefaction parameter significantly enhance both the fluid and volumetric flow rate. Adebawale (2022) studied the impact of second-order slip combined with chemical reaction on the boundary layer flow and heat transfer characteristics of a non-Newtonian nanofluid in a non-Darcy porous medium. The researcher observed that the type second slip parameter significantly affects the outcomes and explored the underlying physical model. Shehu et al. (2022) investigate the magnetohydrodynamic (MHD) slip flow of a viscous reactive fluid in a vertically oriented porous channel subjected to convective heating, employed a perturbation series method and considered the application of suction or injection as a means to control fluid flow within the channel. Okeyode and Waheed (2023) analyzed the second law of thermodynamics in relation to passive control along with heat generation and absorption. This research investigated the combined effects of Arrhenius activation energy and binary chemical reaction in the context of a second-order momentum slip model was carried out by Aaqib et al. (2018). The MHD free convection flow of an exothermic fluid was examined in a vertical channel with Navier slip boundary conditions. Also, Sharada and Shankar (2017) investigated the effects of partial slip and convective boundary conditions on MHD mixed convection flow, both with and without the presence of Joule heating.

The effects of Navier slip, convective cooling, variable viscosity, and suction/injection on the entropy generation rate in an unsteady flow of an incompressible viscous fluid through a channel with permeable walls were examined. It was found out that the lower and upper walls of the channel were subjected to asymmetric convective exchange with the surrounding and allowing for uniform suction/injection in the transverse direction were analyzed by Chinyoka and Makinde (2013) Hamza (2016) studied the steady and unsteady convective flow of an exothermic fluid in a vertical channel with Navier slip conditions and Newtonian heating. The author employed the perturbation series method to solve the steady-state problem and utilized an unconditionally stable and convergent implicit finite difference scheme for the numerical solution of the unsteady problem. The findings indicate that the flow behavior is highly influenced by Newtonian heating, Navier slip, and Arrhenius chemical reaction. Malvandi et al. (2014) considered the effects of slip and convective heat boundary conditions on the steady two-dimensional boundary layer flow of nanofluids over a stretching surface with simultaneous blowing/suction. The study included Navier slip conditions at the surface and considered the Biot number. It was found that, unlike heat transfer rate, the concentration rate s highly sensitive to all parameters. The boundary layer flow over a stretching sheet with convective boundary condition and the effect of slip were observed by Mona and Ebaid (2016). Recently Hamza et al. (2022) investigated the impact of magnetohydrodynamic (MHD) on time-dependent mixed convection flow of an exothermic fluid within a vertical channel. The researcher included convection heating and Navier's slip condition in their analysis. They used the homotopy perturbation method to achieve a steady-state flow and employed a numerical technique to solve the unsteady governing equations. Both steady and unsteady states were described through velocity and temperature profiles. The findings revealed that the Hartmann number decreases momentum profiles, while the fluid temperature and velocity increase with higher thermal Biot numbers and Frank-Kamenetskii parameters. Akbar et al. (2014) the study examined the two-dimensional stagnation-point flow of carbon nanotubes towards a stretching sheet, using water as the base fluid. The analysis considered the effects of slip and convective boundary conditions, employing a homogeneous model. The analysis focused on the steady two-dimensional MHD laminar free convective boundary layer slip flow of an electrically conducting Newtonian nanofluid from a translating stretching sheet in a still fluid, incorporating convective boundary condition was analyzed by Uddin (2014). In view of Alaa and Akil (2020) the study examines the issue of thermal solutal convection in a Navier-Stokes-Voigt fluid when the layer is heated from either above or below, considering the influence of Soret and slip boundary conditions and addressing both linear and nonlinear stability. Additionally, it is noted that linear analysis is sufficiently accurate to predict the onset of convective motion. The study investigated the flow of MHD nanofluid through a porous matrix and convective heating past a permeable, linear stretching sheet, considering the effects of velocity slip, viscous dissipation, Joule heating, and nonlinear thermal radiation was conducted by Nayak et al. (2018). Das et al. (2017) the experiment examined the combined effects of a magnetic field, Navier slip, and convective heating on entropy generation in the flow of a viscous incompressible electrically conducting fluid between two parallel plates. Both the upper and lower plates of the channel were exposed to asymmetric convective heat exchange with the surrounding fluid. It was observed that the plate surface are significant sources of entropy generation number decreases as the magnetic parameter increases. Similarly, the study investigated the impact of Navier slip and other flow parameters on the unsteady flow of a reactive variable viscosity third-grade fluid with symmetric convective heat exchange with the surroundings. The fluid flow involved an exothermic chemical reaction. It was found that increasing the lower wall slip parameter enhances the fluid velocity profiles, while increasing the upper wall slip parameter decreases them due to backflow at the upper channel were examined by Rundura and Makinde (2015). Kaladhar and Komuraiah (2017) investigated the laminar, incompressible free convective Navier slip flow between vertical plates, considering the cross-diffusion effect and first-order chemical reaction. The authors analyze how engineering parameters affect fluid flow quantities and the characteristics of physical properties. Srinivacharya and Bindu (2015) examined the entropy generation in a micropolar fluid flowing through an inclined channel with slip and convective boundary conditions. Torobi and Aziz (2012) and Torobi and Zhang (2014) investigated the impact of convective-radiation boundary condition on the entropy generation rate in an asymmetrically cooled hollow cylinder and cooled homogeneous slabs.

The main purpose of this paper is to modify the work of Hamza et al. (2021) by incorporating the effects of Navier slip and convective heating conditions. The involvement of Navier slip and Newtonian heating conditions at the boundary layer can substantially influence the drag force and the whole flow configuration. Consequently, these actions make the thermal boundary layer to be thicker or thinner depending on the strength of the slip velocity and Newtonian heating cases, hence the motivation for this research. A semi-analytical method is used to solved the modelled governing equations and the impacts of pertinent parameters embedded in the flow problem is graphically illustrated and discussed. Additionally, the physical components of engineering applications in terms of Nusselt number and skin friction are also computed and explained.

MATERIALS AND METHODS

Consider a steady state MHD natural convection flow of an exothermic fluid of Arrhenius kinetic with heat transfer and Navier slip condition in a channel formed by two infinite vertical parallel plates separated by a distance H as shown in Fig. 1. The flow is induced by the convective heating introduced on the lower surface of the channel wall as well as the reactive property of the fluid. Following Hamza et al. (2021) the non-dimensional governing equations under the Boussinesq's Approximation can be written as:

$$v\frac{d^2u'}{dz'^2}+g\beta(T'-T'_0)-\frac{\Delta B_0^2 u'}{\rho}=0 \tag{1}$$

$$\frac{k}{\rho C_p}\frac{d^2u'}{dz'^2}+\frac{QC_0^*A}{\rho C_p}e^{\left(\frac{-E}{RT'}\right)}=0 \tag{2}$$

The boundary condition for the present problem are

$$u'=\gamma^*\frac{du'}{dz'},-k\frac{dT'}{dz'}=h[T_0'-T'],at\,z'=0$$

$$u'=\gamma^*\frac{du'}{dz'},-k\frac{dT'}{dz'}=h[T'-T_0'],as\,z'\to H \tag{3}$$

Where β is the coefficient of thermal expansion, Q is the heat reaction, A is the rate constant, E is the activation energy, R is the universal constant, v is the kinematic viscosity, C_0 is the initial concentration of the reactant species, g is the gravitational force, C_p is the specific heat at constant pressure and k is the thermal conductivity of the fluid, while p is the density of the fluid.

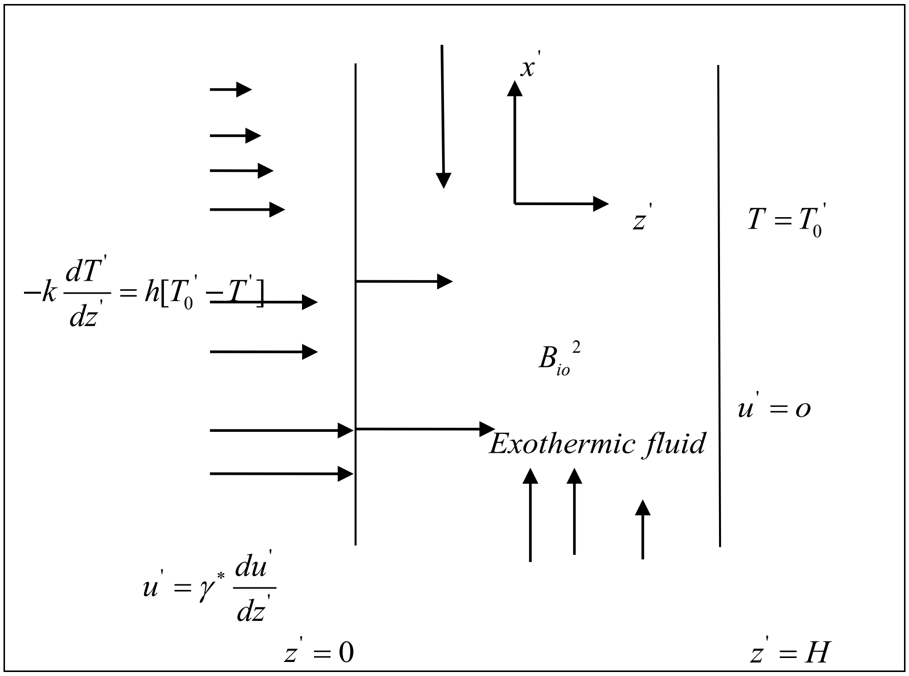

Fig. 1: Geometry of the flow problem

To solve equation (1) and (2), we employ the following dimensionless variables and parameters

$$y = \frac{y'}{H},\ \theta = \frac{E(T'-T_0)}{RT_0^2},\ \varepsilon = \frac{RT_0}{E},\ U = \frac{u'\mu_0 E}{g\beta H^2 RT_0^2},$$

$$\lambda = \frac{QC_0^* AEH^2}{RT_0^2} e^{\left(\frac{-E}{RT_0}\right)},\ \Pr = \frac{\mu_0 \rho Cp}{k},\ \gamma = \frac{\gamma^*}{H},$$

$$\theta_a = \frac{E(T_a - T_0)}{RT_0^2},\ Br = \frac{hH}{k} \tag{4}$$

Using (4), Equations (1)-(3) take the following form:

$$\frac{d^2u}{dz^2} - Ha^2 U + \theta = 0 \tag{5}$$

$$\frac{1}{\Pr}\frac{d^2\theta}{dz^2} + \frac{\lambda}{\Pr} e^{\left(\frac{\theta}{1-\varepsilon\theta}\right)} = 0 \tag{6}$$

The initial and boundary conditions in dimensionless form are:

$$U = \gamma_1 \frac{du}{dz},\ \frac{d\theta}{dz} = B_{i1}[\theta - \theta_a]\ \ at\ \ z = 0$$

$$U = \gamma_2 \frac{du}{dz},\ \frac{d\theta}{dz} = B_{i2}[\theta_a - \theta]\ \ at\ \ z = 1 \tag{7}$$

Where $\gamma, Br, \lambda, \theta\, and\, \varepsilon$ are Navier slip parameter, Biot number, Frank-Kamenetskii parameter, ambient temperature and activation energy parameter.

METHOD OF SOLUTION

The governing equations highlighted in the previous section are nonlinear and exhibit no analytical solutions. The steady state governing equations together with boundary conditions can be written as follows:

$$\frac{d^2U}{dz^2} - Ha^2 U = -\theta \tag{8}$$

$$\frac{d^2\theta}{dz^2} + \lambda e^{\left(\frac{\theta}{1+\varepsilon\theta}\right)} = 0 \tag{9}$$

Subject to the following Boundary conditions

$$U = \gamma_1 \frac{du}{dz},\ \frac{d\theta}{dz} = B_{i1}[\theta - \theta_a]\ \ at\ \ z = 0$$

$$U = \gamma_2 \frac{du}{dz},\ \frac{d\theta}{dz} = B_{i2}[\theta_a - \theta]\ \ at\ \ z = 1 \tag{10}$$

To obtain the approximate solutions (8) and (9) subject to (10), we use regular perturbation method of the form:

$$U = U_0 + \lambda U_1 + \lambda^2 U_2 + \lambda^3 U_3 + 0(\lambda)$$

$$\theta = \theta_0 + \lambda\theta_1 + \lambda^2\theta_2 + \lambda^3\theta_3 + 0(\lambda) \tag{11}$$

Substituting (11) into (8)-(10) and equating the coefficient of like powers of λ the resulting solutions of the momentum and energy balance equations are as follows:

$$U = h_1 z^7 + h_2 z^6 + h_3 z^5 + h_4 z^4 + h_5 z^3 + h_6 z^2 + h_7 z + h_8 \tag{12}$$

$$\theta = k_1 z^6 + k_2 z^5 + k_3 z^4 + k_4 z^3 + k_5 z^2 + k_6 z + k_7 + \theta_a \tag{13}$$

From (12), the steady state skin frictions on the boundaries are as follows:

$$\left.\frac{dU}{dz}\right|_{z=0} = h_7 \tag{14}$$

$$\left.\frac{dU}{dz}\right|_{z=1} = 7h_1 + 6h_2 + 5h_3 + 4h_4 + 3h_5 + 2h_6 + h_7 \qquad (15)$$

Also, from (13) the rates of heat transfer on the boundaries in terms of Nusselt number are calculated as follows:

$$\left.\frac{d\theta}{dz}\right|_{z=0} = k_6 \qquad (16)$$

$$\left.\frac{d\theta}{dz}\right|_{z=1} = 6k_1 + 5k_2 + 4k_3 + 3k_4 + 2k_5 + k_6 \qquad (17)$$

The constants

$h_1, h_2, h_3, h_4, h_5, h_6, h_7,\ k_1, k_2, k_3, k_4, k_5, k_6, k_7, k_8,$

are defined in the appendix section.

RESULTS AND DISCUSSION

In this section, we present the results of the analysis of magnetized chemically reacting fluid influenced by Arrhenius kinetics in the coexistence of Navier slip and convective boundary conditions. The governing parameters dictating the flow system are as follows: Hartmann number (Ha), Biot numbers $(B_{i1}\ and\ B_{i2})$, Navier slip parameters $(\gamma_1\ and\ \gamma_2)$, activation energy (ε), Frank – Kamenetskii parameter (λ). The following values are fixed for the governing parameters. $B_{i1} = B_{i2} = 0.1,\ \gamma_1 = \gamma_2 = 0.01,\ \varepsilon = 0.01,$ $Ha = 0.1,\ \lambda = 0.01,\ \theta_a = 0.1.$

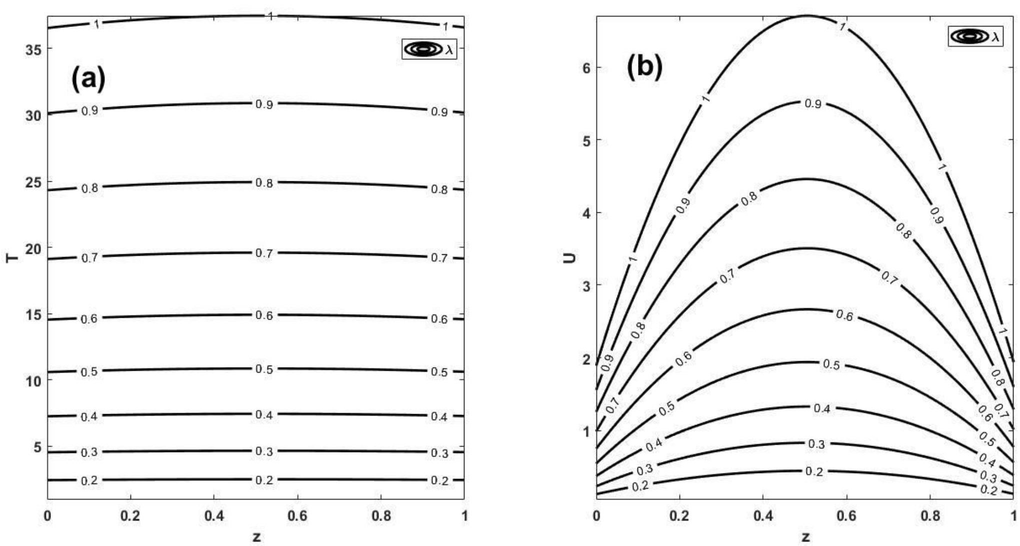

Fig. 2. Effect of λ on Temperature and Velocity profiles

The effect of Frank-Kamenetskii parameter (λ) on both temperature and velocity profiles is showed in Fig 2a and b respectively. From Fig. 2a, it is observed that the fluid temperature increases with increasing value of (λ). In Fig. 2b, we also realized that an increase in the velocity profile is achieved with increasing value of (λ).This is owing to the truth that a growth in λ, which basically denotes the viscous heating parameter, corresponds to an increase in the chemical reaction's strength. Naturally speaking, these results make sense, as raising the heat source parameter indicates more heat generation from the region's surface, making the fluid to accelerate faster, thereby resulting in high performance of the system. There are many thermal applications where this result can find relevance which include power generation, space heating, cooking and industrial processes, to mention a few. This result corroborates those obtained by Obalalu *et al*. (2021) and Hamza (2013).

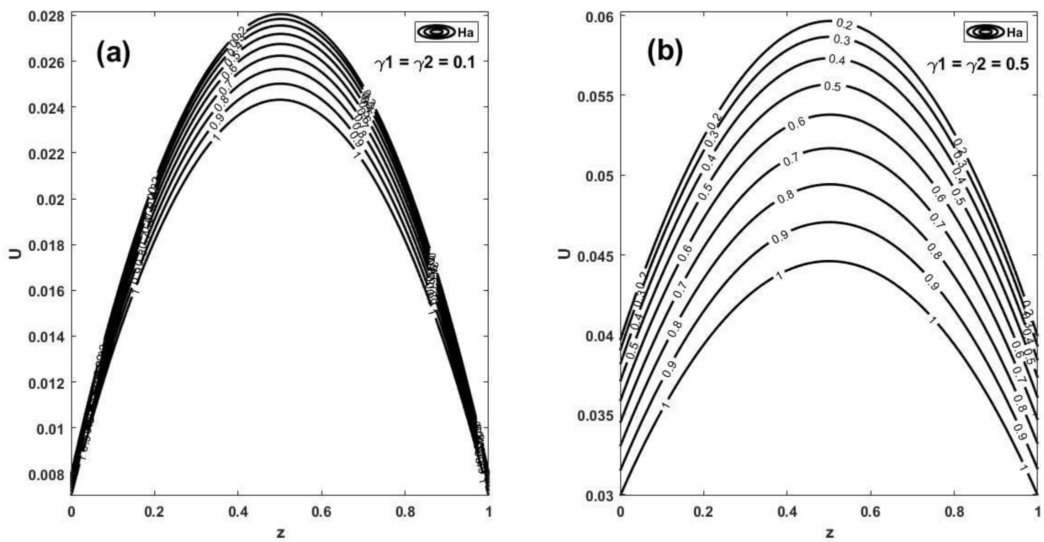

Fig. 3. Effect of Ha on velocity profiles

The influence of Hartmann number (Ha) and Navier slip parameters $(\gamma_1 \; and \; \gamma_2)$ on velocity profile is shown in Fig. 3a and b respectively. It is obvious that the pattern demonstrates a decrease in the fluid flow as the magnetic field intensity increases. This is physically true, owing to the Lorentz force, which appears when a magnetic field imposes itself on an electrically conducting fluid and a drag force is created. Due to this force, fluid movement dwindles near the plate; all other forces, including the Lorentz force, decays as a result when the fluid comes to rest.Also, Fig 3b revealed that the fluid velocity decreases with increasing value of $(\gamma_1 \; and \; \gamma_2)$.

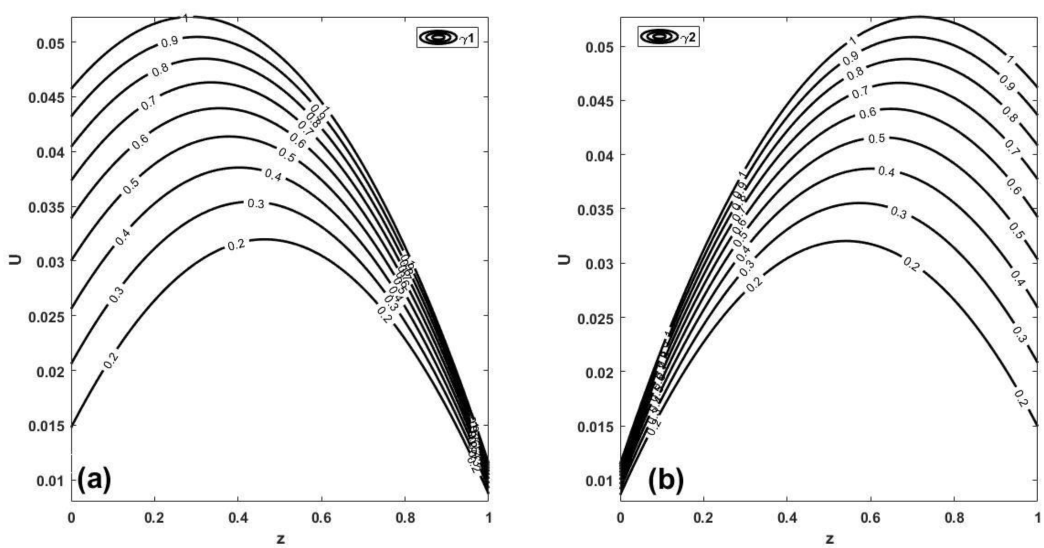

Fig.4. Effect of slip parameter on velocity profile

Fig. 4a and b explain the effect of Navier slip parameters ((γ_1) and (γ_2))respectively on the velocity profile. Fig. 4a shows that when slip parameters $(\gamma_1 = \gamma_2 = 0.5)$is higher, the fluid velocity decreases. On the other hand, Fig. 4b shows a significant increase on the velocity profile for growing value of (γ_2). This is so because higher values of (γ_1) and (γ_2) correspond to an increase in the reaction and slipperiness of the plate surfaces, leading to a faster movement of the fluid.

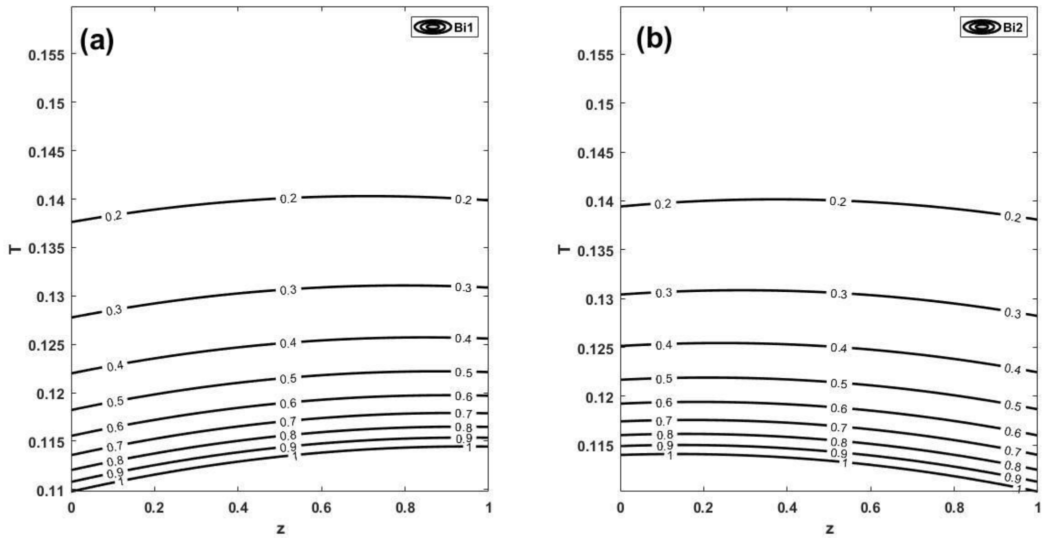

Fig.5. Effect of Biot number on Temperature profile.

Fig. 5a and b show the effect of Biot number on temperature profile. Fig. 5a confirms that there is an obvious retardation in thetemperature profile for greater value of $\left(B_{I1}\right)$. Also, Fig. 5b shows a significant decrease on the temperature profile with increasing value of $\left(B_{I2}\right)$.Similarly,

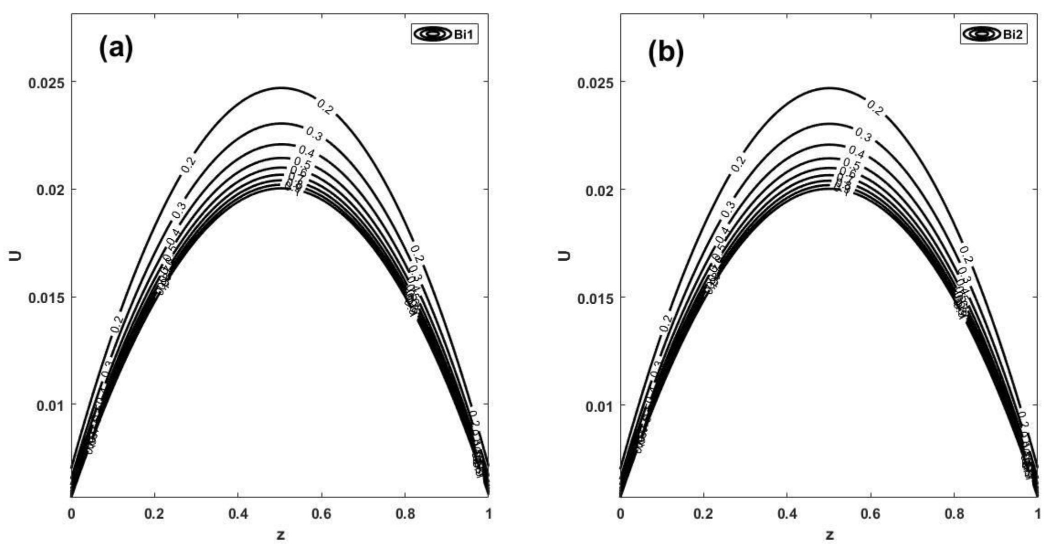

Fig.6. Effect of Biot number on velocity profile

Fig. 6a and b describe the effect of Biot number on velocity profile. Fig. 6a shows that there is a dramatic lowering on velocity profile when the value of $\left(B_{I1}\right)$ is raised. Also, Fig. 6b demonstrates a drastic decline on the velocity profile with rising values of $\left(B_{I2}\right)$.

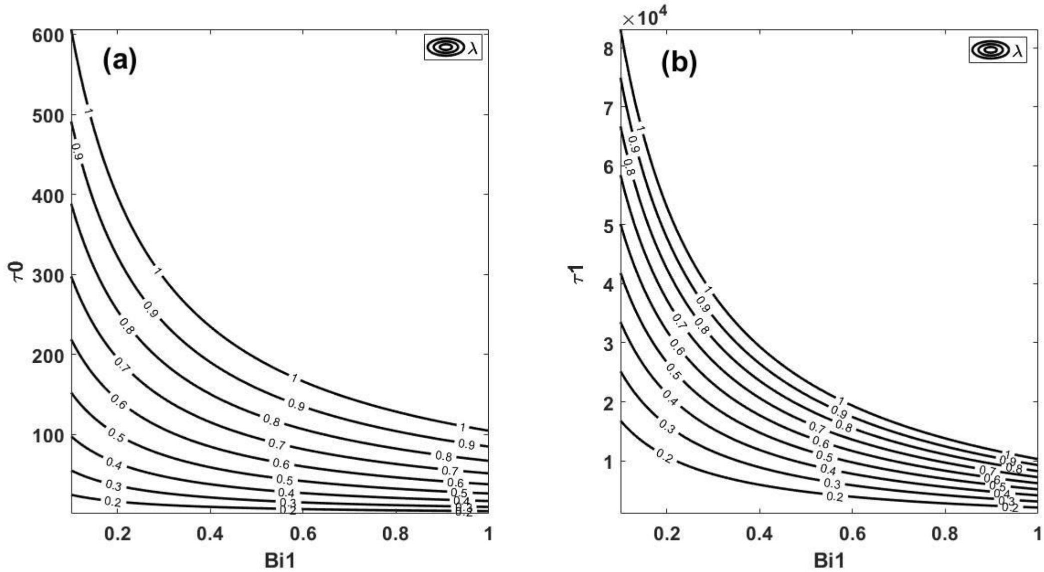

Fig.7. Effect of skin friction on Bi1 for different values of λ

Fig. 7a and b illustrate the shear stress against $\left(B_{I1}\right)$ for different values of $\left(\lambda\right)$. From these figures, it is evident that there is a substantial rise in skin friction at both $\left(\tau_0\right)$ *and* $\left(\tau_1\right)$ when $\left(\lambda\right)$ increases.

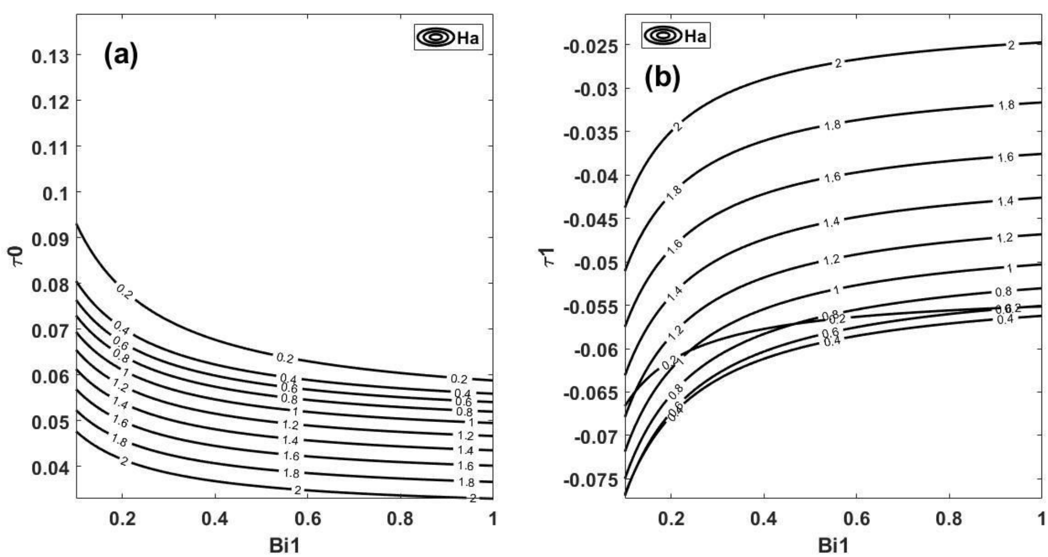

Fig.8. Influence of skin friction against Bi1 for different values of Ha

With the help of Fig. 8a and b, the variation of the skin friction versus $\left(B_{I1}\right)$ for different values of $\left(Ha\right)$ is displayed. A diminishing tendency observed in the frictional force $\left(\tau_0\right)$ with growing level of $\left(Ha\right)$. Contrarily, Fig. 8b demonstrates an opposite behavior in the skin friction $\left(\tau_1\right)$ for mounting value of $\left(Ha\right)$.

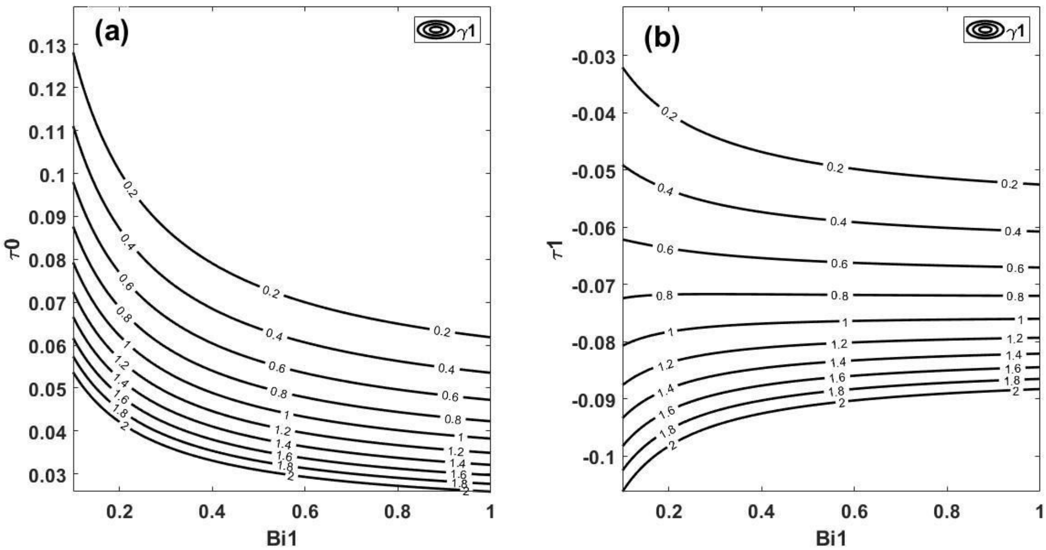

Fig.9. Effect of skin friction against Bi1 for different values of slip parameter

The function of Biot number Bi1 on the skin friction is portrayed in Fig. 9a and b. It is revealed that,for growing values of slip parameter, demonstrate similar trends at the both plates.

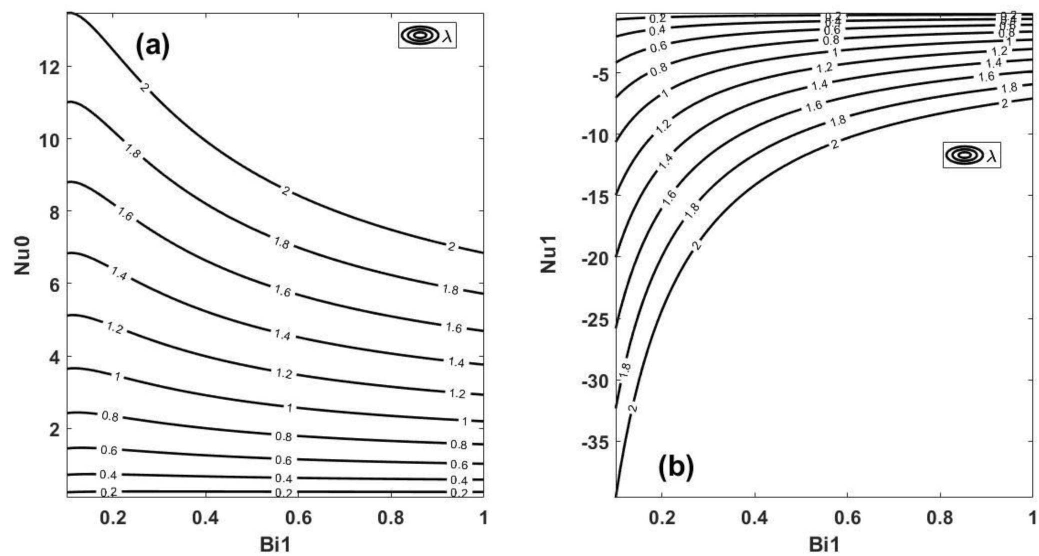

Fig.10. Influence of Nusselt number against Bi1 for different values of chemical reacting parameter

Fig. 10 is plotted to showcase the impact of Bi1 in the involvement of chemically reacting parameter on the rate of heat transfer. It is shown from these figures that, at the lower plate, the effect of increasing Bi1 is to improve the heat transfer rate while the counter attribute is recorded at the upper plate.

CONCLUSION

In this article, we have considered the analytical investigation of magnetized chemically reactive fluid with Arrhenius-controlheat transfer affected by Navier slip and convective boundary conditions. The approximate solution of the problem is obtained using regular perturbation method. Illustrative graphs explaining the flow variations of the pertinent parameters is plotted and the results are discussed in line with real life applications. These kinds of studies are useful since Navier slip and Newtonian heating contribute to the thinness or thickness of the thermal boundary layer, which in turn, significantly affect the skin friction and the overall flow patterns. Summary of the major findings from this work are highlighted below;

i. The effect of increasing the value of λ is seen to drastically enhance both the temperature and the velocity gradients respectively.
ii. Uplifting the values of slip parameter ($\gamma_1 = \gamma_2$) respectively establish an opposite but ascending pattern in the fluid velocity.
iii. The temperature and velocity distributions exhibit similar descending behavior for greater values of Biot numbers (B_{i1} *and* B_{i2}) respectively.
iv. It is concluded that raising the levels of λ against Bi1 portray identical increasing effect
v. A weakening drag force is achieved at the lower and upper plates by raising the value of slip parameter
vi. The rate of heat transfer is enhanced for higher values of λ at the lower plate (z=0) whereas a counter behavior is noticed at the upper plate (z=1).

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Appendix A

$$a_1=\frac{1}{2}-\varepsilon,\ A_2=\theta_a,\ A_1=0,\ \theta_0=\theta_a,\ a_2=1+\theta_a+a_1\theta_a^2,\ a_3=1/2(1+\theta_a+a_1\theta^2{}_a)\ A_3=B_{i1}A_4,$$

$$A_4=\frac{a_2+B_{i2}a_3}{(B_{i1}+B_{i1}B_{i2}+B_{i2})},\ k_1=-1-\theta_a{}^2-a_1\theta_a{}^2,\ \theta_1=\frac{k_1z^2}{2}+A_3z+A_4,\ k_2=\frac{-k_1}{2}-a_1k_1\theta_a$$

$$k_3=-A_3-2a_1A_3\theta_a,\ k_4=-A_4-2a_1A_4\theta_a,\ k_5=B_{i1}k_6,\ k_6=\frac{-k_9}{k_{10}},\ k_7=\frac{k_2}{3}+\frac{k_3}{2}+k_4\ k_8=\frac{k_2}{12}+\frac{k_3}{6}+\frac{k_4}{2},$$

$$k_9=B_{i2}k_8+k_7\ k_{10}=B_{i1}+B_{i2}B_{i1}+B_{i2}\ \theta_2=-\frac{b_2z^4}{12}-\frac{b_3z^3}{6}-\frac{k_4z^2}{2}+k_5z+k_6,\ D_3=\frac{\theta_a}{Ha^2}$$

$$D_1=\gamma HaD_2-D_3\ m_1=\cosh(Ha)-\gamma_2Ha\sinh(Ha)\ m_2=\sinh(Ha)-\gamma_2Ha\cosh(Ha)$$

$$D_2=\frac{D_3(m_1-1)}{m_1\gamma_1Ha+m_2}\ U_0=D_1\cosh(Haz)+D_2\sinh(Haz)+D_3\ D_5=\frac{D_{10}-\gamma_1D_{11}D_7+D_8D_{11}}{\gamma_1D_{11}Ha+D_{12}}$$

$$D_6=\frac{k_1}{2Ha^2}\ D_7=\frac{A_3}{Ha^2}\ D_8=\frac{A_4+2D_6}{Ha^2}\ D_4=\gamma_1HaD_5+\gamma_1D_7-D_8\ D_{11}=\cosh(Ha)-\gamma_2Ha\sinh(Ha)$$

$$D_{12}=\sinh(Ha)-\gamma_2Ha\cosh(Ha)\ U_1=D_4\cosh(Haz)+D_5\sinh(Haz)+,D_6z^2+D_7z+D_8$$

$$f_1=\gamma_1\left(Haf_2+f_6\right)-f_7\ f_3=\frac{k_2}{12Ha^2}\ f_4=\frac{k_3}{6Ha^2}\ f_5=\frac{k_4+12f_3}{2Ha^2}\ f_6=\frac{k_5+6f_4}{Ha^2}\ f_7=\frac{k_6+2f_5}{Ha^2}$$

$$L_1=\cosh(Ha)-\gamma_2Ha\sinh(Ha)\ L_2=\sinh(Ha)-\gamma_2Ha\cosh(Ha)$$

$$L_3=-f_3-f_4-f_5-f_6-f_7+\gamma_2\left(4f_3+3f_4+2f_5+f_6\right)\ f_2=\frac{L_3-L_1(\gamma_1f_6+f_7)}{L_1\gamma_1Ha+L_2}$$

$$U_2=f_1\cosh(Haz)+f_2\sinh(Haz)+f_3z^4+f_4z^3+f_5z^2+f_6z+f_7$$