Journal of Basics and Applied Sciences Research (JOBASR)
ISSN (print): 3026-9091, ISSN (online): 1597-9962
Volume 3(2) March 2025
DOI: https://dx.doi.org/10.4314/jobasr.v3i2.6

Wave Propagation Patterns for the (2+1) Dimensional Modified Complex KDV System via New Extended Direct Algebra Method

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ABSTRACT
In this article, we derive various exact solutions and patterns for the complex modified Korteweg–De Vries system of equation (cmKdV) with a generalized innovative extended direct algebra method. The Korteweg-De Vries system exhibits the scientific dynamics of water particles at the surface and beyond the surface level. The system also has applications in ferromagnetic materials, nonlinear optics, and solitons theory. The innovative direct algebra method is applied to obtain dark, multiple, singular, breather and bright wave patterns. This method also provides staggering wave solutions for the complex modified Kortweg-De Vries system in the form of hyperbolic and trigonometric functions. These recovered solutions for the considered model and are more efficient, concise and general than the extant ones. The wave patterns are properly explained with 2-D and 3-D graphs to elucidate wave behaviour for some selected solutions derived for the system. Lastly, the solutions in this work will greatly advance various fields of application of the Kortweg-De Vries equation like optical fibres, ferromagnetic materials, nonlinear optics, signal processing, water waves, plasma physics, soliton theory, string theory and other contemporary sciences.

Keywords:
Modified complex Kortweg-De Vries system,
wave patterns,
extended direct algebra method,
water particles.

INTRODUCTION

In recent years, researchers have put considerable efforts into nonlinear waves at oceanic surfaces. The phenomena of nonlinear waves significantly contribute to tsunami waves, ocean engineering, mechanics, Control theory, biology, plasma physics, the communications industry, coastal engineering, fluid dynamics, physics, chemistry, and so forth. In many engineering and science research areas, nonlinear wave propagations are vital in the multidisciplinary sciences (Sabi'u, Das & Razazadeh, 2022). In recent days, researchers are investigating unique solutions for nonlinear partial differential equations (NPDEs) by utilizing distinct techniques. In specific, for the solitary wave solutions, many powerful techniques have been offered, like the Ricatti equation method (Ibrahim, Sabiu & Gambo, 2024), tanh-coth, sine-cosine and Kudryashov methods (Shaikova, Kutum & Myrzakulov, 2022), the Sadar subequation method (Muhammad, Sabi'u, Salahshour, & Rezazadeh, 2024) the tanh technique (Jibril & Gadu, 2019), the Hirota approach (Akinyemi, Senol & Iyiola, 2021), exp-function technique (Khater, Akinyemi, Elagan & El-Shorbagy, 2021), the Jacobi elliptic function technique (Houwe & Abbagari, 2021), as (Razazadeh, 2018), stated in Darboux transformation approach and so on in providing the unified framework. Most of these methods are successfully applied to obtain solitary wave solutions to the wide range of nonlinear evolution equations, for example, the application of the auxiliary equation method to Biswas–Arshed equation (Razazadeh, 2018), the application of the new direct algebra methods on the Triki-Biswas equation (Rahman, Karaca & Baleanu, 2024), The application of the generalized Riccation equation and direct algebra on the Chavy-Waddy-Kolokolnikov model for bacterial colonies , the application of modified hyperbolic function method to the generalized (2+1)-dimensional nonlinear wave equation (Shaikova, Kutum & Myrzakulov, 2022), the application of exponential function expansion and Kudryashov methods (Rahman, Karaca & Baleanu, 2024), the application of the (G'/G)-expansion method to the Mikhailov–Novikov–Wang equation (Ibrahim, Sabiu & Gambo, 2024), the application of the extended trial equation method to the perturbed Boussinesq equation with power law nonlinearity (Shaikova, Kutum & Myrzakulov, 2022), the application of the sine-cosine method on the unstable and hyperbolic nonlinear Schr¨odinger equations (Akinyemi, Senol & Iyiola, 2021), etc. In this research we will investigate different wave patterns and solitary wave solutions to the complex modified Kortweg-De Vries (cmKdV) system (Razazadeh, 2022) using the new extended direct algebra method. This method has so much significance as it provides a systematic approach to finding exact solutions to NPDEs which is crucial for understanding complex phenomena, and the method reduces complex NPDEs to simpler algebraic equations, making it easier to analyze and solve them. The cmKdV system is as follows:

$$\begin{aligned} H_t + H_{xxy} + (HW)_x + iHV = 0, \\ V_x + 2i\tau\left(H^* H_{xy} - H^*_{xy} H\right) = 0, \\ W_x - 2\tau(|H|^2)_y = 0. \end{aligned} \tag{1}$$

$H(x,y,t)$ is a complex-valued function, $H^*(x,y,t)$ is the conjugate complex function, while $V(x,y,t)$, and $W(x,y,t)$ are real-valued functions, x, y are spatial coordinate, t is a temporal coordinate, $i$is complex and $\tau = \pm 1$. Furthermore, some bright, periodic and dark soliton solutions of Eq. (1) are acquired for the system Eq. (1) by utilising three different approaches: tanh-coth, sine-cosine and the Kudryashov methods in (Ibrahim, Sabiu & Gambo, 2024). Other techniques applied to solve the cmKdV system Eq. (1) are the Sardar sub-equation technique (Yuan, 2021), the improved Ricatti approach (Shaikhova, Kutum & Myrzakulov, 2022), and the planar dynamical system method (Sabi'u, Das & Razazadeh, 2022). This research will use the innovative direct algebra approach to present unique travelling wave solutions for the Eq. (1). The new extended direct algebra technique is efficient for deriving exact solutions for the NPDEs. Using the travelling wave technique, this approach transforms NPDEs into nonlinear ordinary differential equations. The method is straightforward and gives numerous exact solutions to the NPDEs that can be characterised as periodic, hyperbolic, exponential and rational function solutions, see (Razazadeh, 2018) for more details about the new extended direct algebra method. Additionally, we give a brief insight into the lax-pair integrability test for the considered model.

The structure of the paper is composed of the following sections: section 1, Lax pair analysis for the cmKdV system were presented. In section 2, the main procedure associated with the new extended direct algebra method was depicted. In section 3, the exact solutions of the considered model are displayed. In section 4, the results and discussion of the findings are exhibited. section 5, completes the paper with some further research directions.

MATERIALS AND METHODS

Lax pair analysis of the modified complex KdV model

This section will give a brief insight into the lax-pair integrability test for the considered model. The proportional Lax pair analysis of Eq. (1) as described in (Shaikhova, Kutum & Myrzakulov, 2022) by letting

$$\Phi_x = B\Phi, \qquad \Phi_t = 4\lambda^2\Phi_y + A\Phi, \tag{2}$$

with

$$B = \lambda K + B_0, \qquad A = \lambda A_1 + A_0, \tag{3}$$

where:

$$\Phi = \begin{bmatrix} \pi_1(\lambda, x, y, t) \\ \pi_2(\lambda, x, y, t) \end{bmatrix}, \quad K = \begin{bmatrix} -i & 0 \\ 0 & i \end{bmatrix} \tag{4}$$

$$A_0 = \begin{bmatrix} -\frac{iV}{2} & -H_{xy} - W_y \\ G_{xy} + WG & \frac{iV}{2} \end{bmatrix}, \quad B_0 = \begin{bmatrix} 0 & H \\ -G & 0 \end{bmatrix}, \quad A_1 = \begin{bmatrix} iW & 2iH_y \\ 2iG_y & -iW \end{bmatrix} \tag{5}$$

and $\lambda \in$C.

The compatibility condition

$$B_t - A_x - BA + AB - 4\lambda^2 B_y = 0, \tag{6}$$

derived the (2+1)-dimensional combined complex modified KdV equations as shown below:

$$\begin{cases} H_t + H_{xxy} + iVH + (WH)_x = 0 \\ G_t + G_{xxy} - iVG + (WG)_x = 0, \\ V_x + 2i(GH_{xy} - G_{xy}H) = 0, \\ W_x - 2(HG)_y = 0. \end{cases} \tag{7}$$

By setting $G = \tau H^*$, the Eq. (7) will be simplified to the complex modified KdV equation shown in Eq. (1). This shows that the considered model is integrable and can be solved via integrable approaches. In this research, we employ the innovative extended direct algebra approach to acquire the exact solutions for the cmKdV equation.

Illustration of the new extended direct algebra method

We will elaborate on the key stages of the new extended direct algebra method for determining the exact solutions of NPDEs.

Step I:

Suppose that the given NPDE for $M(x,y,t)$ is in the format of:

$$Q^{NPDE}(M, Mx, Mt, MxxMtt, ...) = 0 \quad (8)$$

by utilizing this transformation

$$M(x,y,t) = M(\zeta), \quad \zeta = x + y + \alpha t, \quad (9)$$

we get the resulting ordinary differential equation (ODE)

$$Q^{NODE}(\alpha M(\zeta), M'(\zeta), M''(\zeta), ...) = 0 \quad (10)$$

Step II:

Let us assume that the exact solution of the Eq. (10) can be described as a polynomial function in $Q(\zeta)$ as shown below:

$$M(\zeta) = \sum_{i=0}^{K} b_i Q^i(\zeta), \quad b_K \neq 0 \quad (11)$$

Where $b_i (0 \leq i \leq K)$ are the coefficients of constants to be determined subsequently and the function $Q(\zeta)$ satisfied the NODE in this format:

$$Q'(\zeta) = ln(a)(m_1 + m_2 Q(\zeta) + m_3 Q(\zeta)^2) \quad (12)$$

Step III:

To determine the value of the integer K, we balance the highest order derivative with the nonlinear highest term in the Eq. (10). Plugging Eq. (12) and Eq. (11) into Eq. (10) produces an equation in powers of $Q(\zeta)$ in Eq. (10) we then gather all the coefficients of powers of $Q^i (i = 0,1,2,3,\cdots,K)$ in the deduced equation where these coefficients must be set to zero and yield a set of simple algebraic equations containing the constants b_i. Then it can be solved by employing a computer programming tool, like Maple, Mathematica, or Matlab. See (Razazadeh, 2018) for more details on the extended direct algebra method.

Exact solutions of the (2+1) nonlinear modified complex KdV system

Eq. (1) has to be reduced to NODE by taking the wave transformation.

$$H(x,y,t) = e^{i(\omega x + \vartheta y + \varrho t)} M(x,y,t), \quad (13)$$

where the parameters ω, ϑ and ϱ are constants and real and $M(x,y,t)$ is also a real valued function, Eq. (1) are induced to the system of equations as follows:

$$M_t - 2\omega\vartheta M_x - \omega^2 M_y + M_{xxy} + M_x w + M w_x + i((\varrho - \omega^2\vartheta)M + 2\omega M_{xy} + \omega M_{xx} + \omega MW + MV) = 0 \quad (14)$$

$$V_x - 4\tau(\omega M M_x + \vartheta M M_y) = 0, \quad (15)$$

$$W_x - 2\tau(M^2)_y = 0. \quad (16)$$

Plugging the following transformation

$M(x,y,t) = M(\zeta) = M(x + y + \alpha t)$,
$W(x,y,t) = W(\zeta) = W(x + y + \alpha t)$, $V(x,y,t) = V(\zeta) = V(x + y + \alpha t)$,
into Eq. (14)-Eq. (16) we obtain that

$$(\alpha - 2\omega\vartheta - \omega^2)M' + M''' + M'W + MW' + i((\varrho - \omega^2\vartheta)M + (2\omega + \vartheta)M'' + \omega MW + MV) = 0, \quad (17)$$

$$V' - 4\tau(\omega + \vartheta)MM' = 0, \quad (18)$$

$$W' - 2\tau(M^2)' = 0 \quad (19)$$

where $M := M(\zeta)$, $W := W(\zeta)$, and $V := V(\zeta)$. Now, Taking the integral on both sides of Eq. (18)-Eq. (19) with respect to ζ and by setting integral constants to zero, we obtain:

$$V = 2\tau(\omega + \vartheta)M^2, \; W = 2\tau M^2. \quad (20)$$

By inserting Eq. (20) into Eq. (17), we obtained the ODE as follows:

$$(\alpha - 2\omega\vartheta - \omega^2)M' + M''' + 2\tau(M^3)' + i((\varrho - \omega^2\vartheta)M + (2\omega + \vartheta)M'' + 2\tau(2\omega + \vartheta)M^3) = 0. \quad (21)$$

Separating the real part and imaginary part in Eq. (21), the following ODEs were obtained:

$$(\alpha - 2\omega\vartheta - \omega^2)M' + M''' + 2\tau(M^3)' = 0, \quad (22)$$

$$\frac{(\varrho - \omega^2\vartheta)}{(2\omega + \vartheta)}M + M'' + 2\tau M^3 = 0 \quad (23)$$

Taking the integral on both sides of Eq. (22) with respect to ζ, we obtained

$$(\alpha - 2\omega\vartheta - \omega^2)M + M'' + 2\tau M^3 = R \quad (24)$$

where R is the integration constant. The Eq. (23) and Eq. (24) are equivalent if:

$$\alpha - 2\omega\vartheta - \omega^2 = \frac{(\varrho - \omega^2\vartheta)}{(2\omega + \vartheta)}, \quad R = 0 \quad (25)$$

utilising condition Eq. (25), we get

$$\alpha = 2\omega\vartheta + \omega^2 + \frac{\varrho - \omega^2\vartheta}{2\omega + \vartheta} \quad (26)$$

We rewrite equation Eq. (23) as

$$M'' + \frac{(\varrho - \omega^2\vartheta)}{(2\omega + \vartheta)}M + 2\tau M^3 = 0 \quad (27)$$

We solve equation Eq. (27) using the innovative extended direct algebra approach in the next section below.

The application of the new extended direct algebra method

We are going to employ the innovative extended direct algebra technique to find exact solutions of Eq. (27) in this section.By using the homogeneous balance technique into Eq. (27), we get $K = 1$. The exact solution of Eq. (27) can be obtained by utilising

$$M = \sum_{i=0}^{K} b_i Q^i(\zeta),$$
$$b_K \neq 0, \text{ for } i = 0, 1, 2, 3, ...K, \quad (28)$$

This implies that:

$$M(\zeta) = b_0 + b_1 Q(\zeta) \quad (29)$$

be the solution of Eq. (27) using the new extended direct algebra method steps with b_0 and b_1 to be determined later. It is not difficult to see that if we differentiate Eq. (29) we have

$$M(\zeta)' = b_1 \left(\frac{dQ(\zeta)}{d\zeta} \right) \quad (30)$$

Now, substituting Eq. (30) and

$$\frac{d}{d\zeta} Q(\zeta) = \ln(a) \left(m_1 + m_2 Q(\zeta) + m_3 Q(\zeta)^2 \right) \quad (31)$$

into Eq. (27) and setting all the coefficients of powers $Q(\zeta)$ to zero, we obtain:

For:

$$Q(\zeta)^0 : \quad \frac{2m_1 \left(\frac{a+1}{2b}\right) m_2 b_1 (\ln(a))^2 - b_0 \left(-4\left(\frac{a+1}{2b}\right)\eta {b_0}^2 + a^2 b - w\right)}{2a+b} \quad (32)$$

For:

$$Q(\zeta) : \quad 2b_1 \ln(a)^2 m_1 m_3 + b_1 \ln(a)^2 m_3^2 + \frac{(-\omega^2 \vartheta + \varrho)}{2\omega + \vartheta} b_1 + 6\eta b_0^2 b_1 = 0 \quad (33)$$

For:

$$Q(\zeta)^2 : \quad 3b_1 \ln(a)^2 m_2 m_3 + 6\eta b_0 b_1^2 = 0 \quad (34)$$

For:

$$Q(\zeta)^3 : \quad 2b_1 \ln(a)^2 m_3^2 + 2\eta b_1^3 = 0 \quad (35)$$

Solving the above system of equations for b_0, b_1, m_1, m_2 and m_3, with Maple's support, we generated the obtained results as presented under the results and discussion.

RESULTS AND DISCUSSION

This section gives various wave propagation patterns for the solution of the (2+1)-dimensional cmKdV system via the new extended direct algebra method. For this sake, the wave patterns are properly explained with 2-D and 3-D graphs to elucidate wave behaviour for some selected solutions derived for the system. See the figures below:

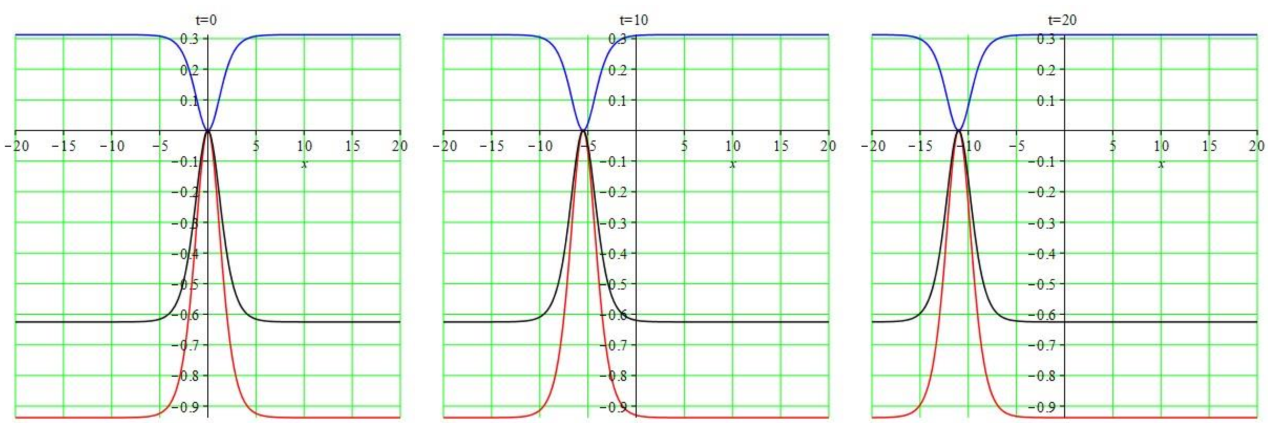

Fig. 1a Fig. 1b Fig. 1c

Figure 1: The 2D propagation for $|H_1(x,y,t)|^2$ with **blue line** indicating **singular-wave soliton**; $V_1(x,y,t)$ with **red line** representing **singular-wave soliton** and $W_1(x,y,t)$ with **black line** showing **singular-wave soliton** for $t = 0$, 10 and 20 seconds.

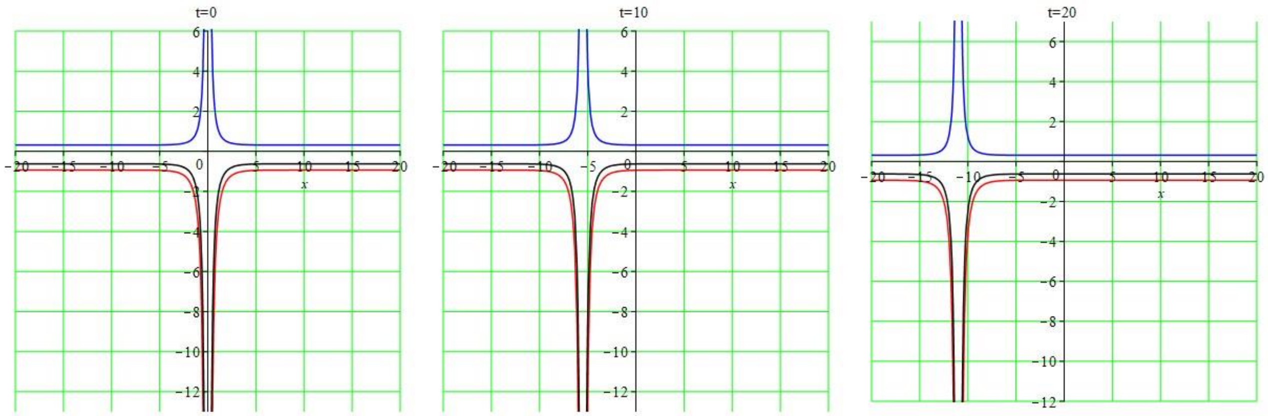

Figure 2: The 2D propagation for $|H_2(x,y,t)|^2$ with **blue line** indicating **bright-wave soliton**; $V_2(x,y,t)$ with **red line** representing **dark-wave soliton** and $W_2(x,y,t)$ with **black line** showing **dark-wave soliton** for t = 0, 10 and 20 seconds.

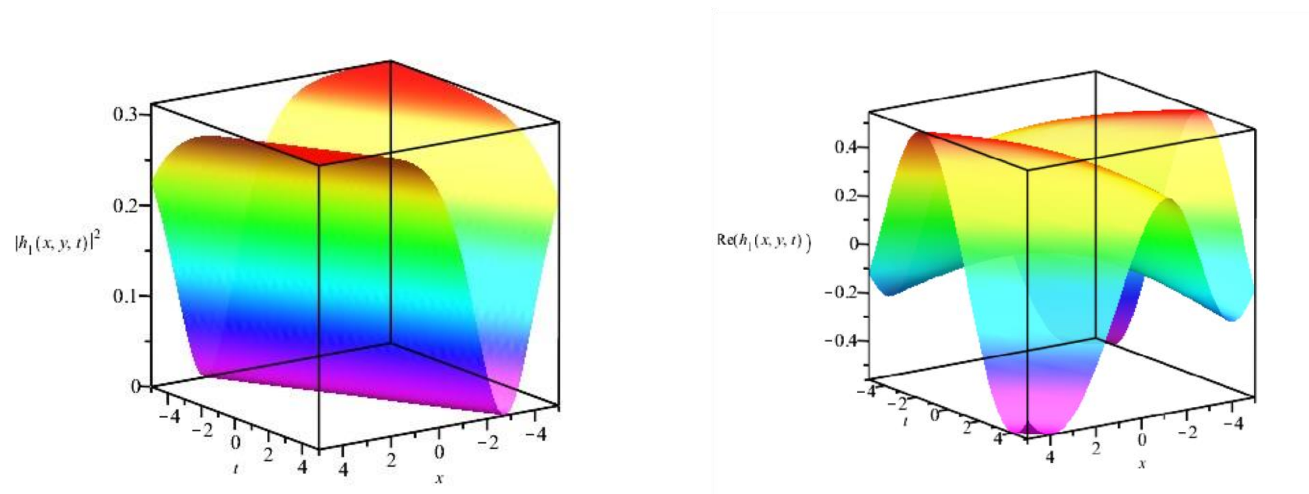

Figure 3: The 3D wave pattern for $|h_1(x,y,t)|^2$ showing **dark-wave soliton** profile and the Re($h_1(x,y,t)$) indicating **multiple-wave soliton** profile for $p = q = a = \omega = 0.5$, $m_3 = \eta = \vartheta = 1$, $\varrho = 1.5$, and $y = 0$.

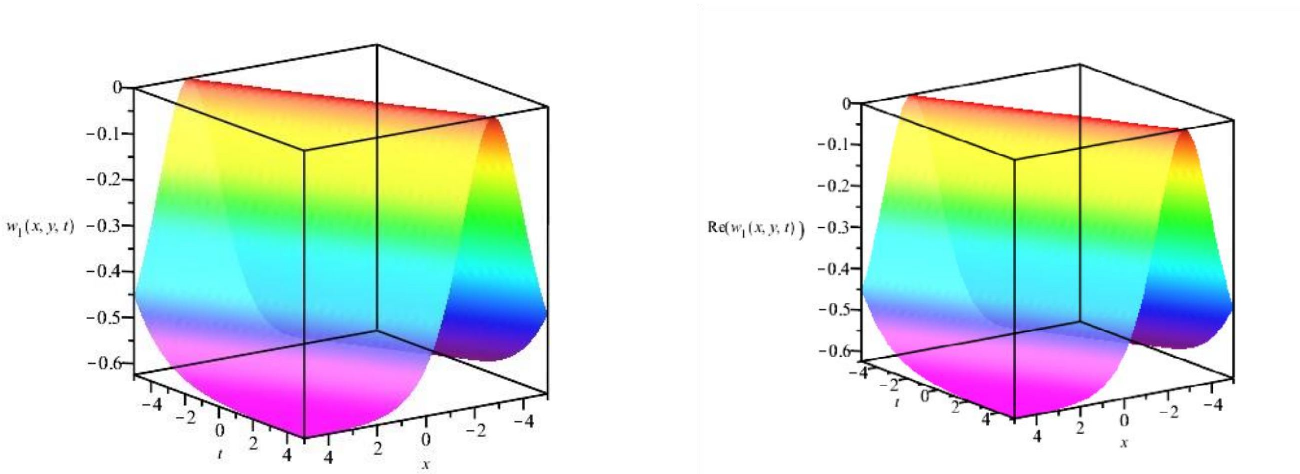

Figure 4: The 3D wave pattern for $V_1(x,y,t)$ and the $\mathrm{Re}(V_1(x,y,t))$ showing the **bright-wave solitons** profiles for $p = q = a = \omega = 0.5$, $m_3 = \eta = \vartheta = 1$, $\varrho = 1.5$, and $y = 0$.

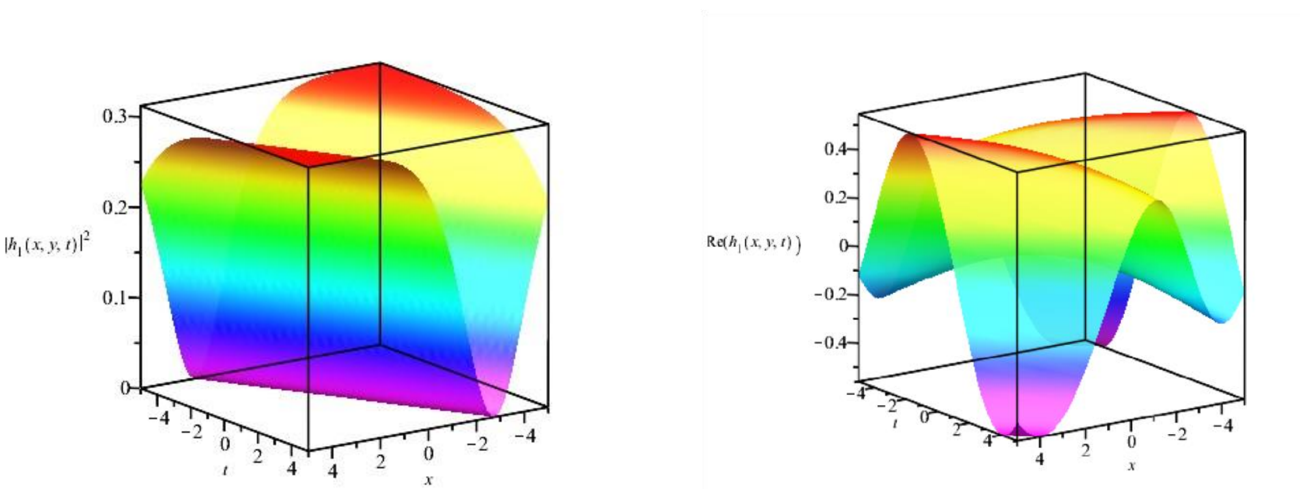

Figure 5: The 3D wave pattern for $W_1(x,y,t)$ and the $\mathrm{Re}(W_1(x,y,t))$ showing **bright-wave solitons** profiles for $p = q = a = \omega = 0.5$, $m_3 = \eta = \vartheta = 1$, $\varrho = 1.5$, and $y = 0$.

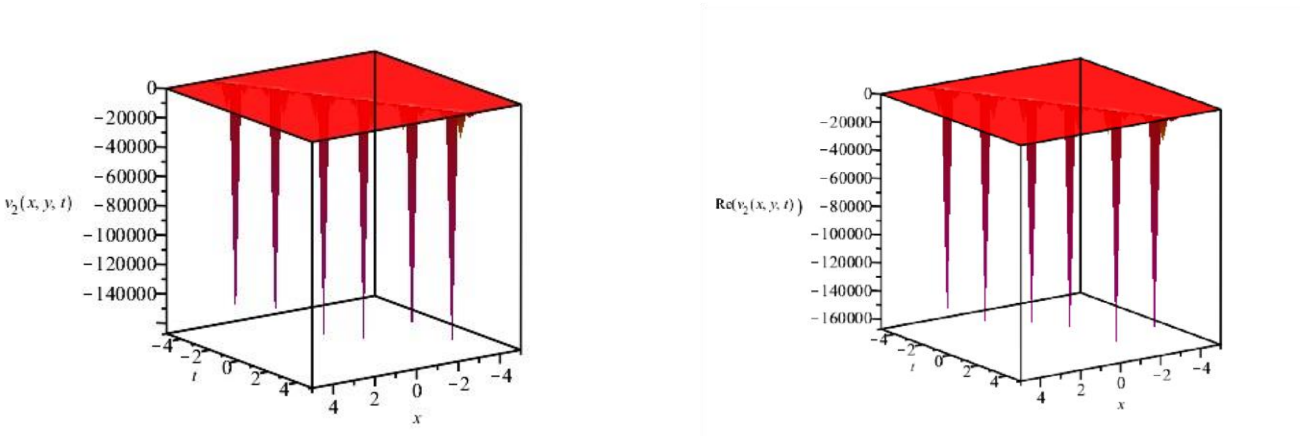

Figure 6: The 3D wave pattern for $|H_2(x,y,t)|^2$ and the $\mathrm{Re}(H_2(x,y,t))$ showing **breather-wave soliton** profiles for $p = q = a = \omega = 0.5$, $m_3 = \eta = \vartheta = 1$, $\varrho = 1.5$, and $y = 0$.

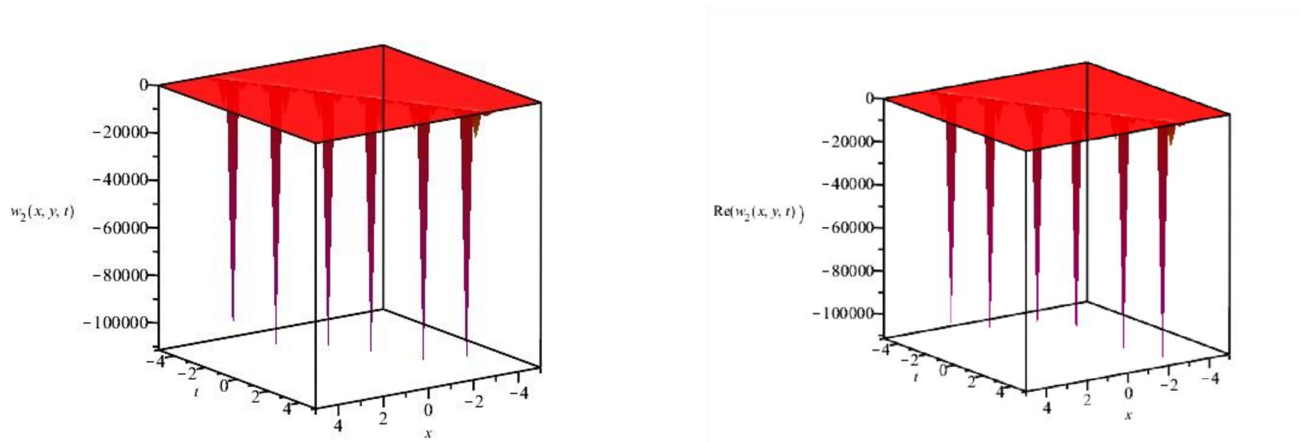

Figure 7: The 3D wave pattern for $V_2(x,y,t)$ and the $\mathrm{Re}(V_2(x,y,t))$ showing **breather-like solitons** profiles for $p = q = a = \omega = 0.5$, $m_3 = \eta = \vartheta = 1$, $\varrho = 1.5$, and $y = 0$.

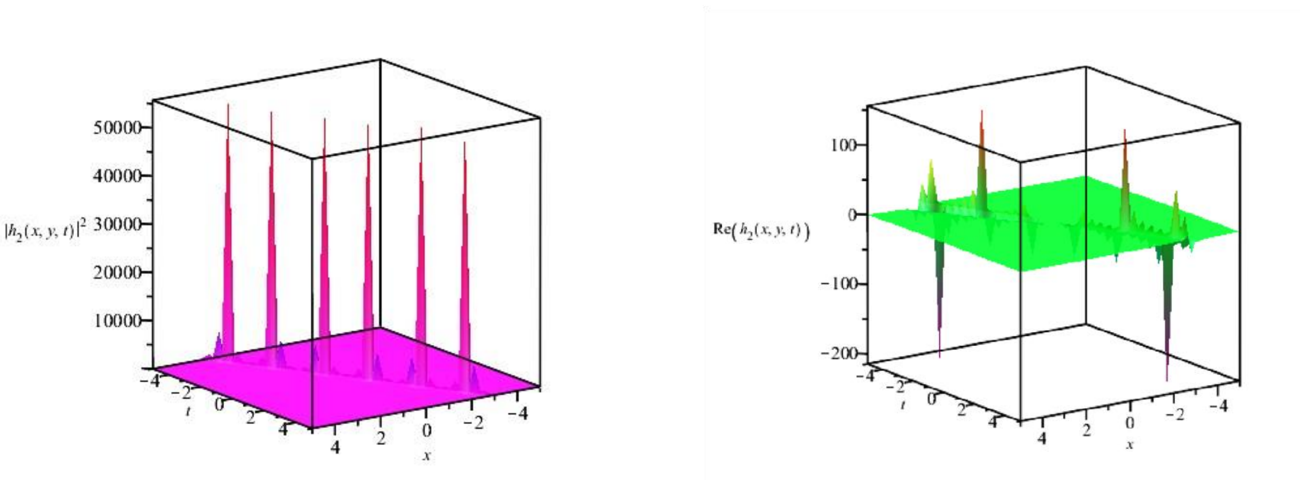

Figure 8: The 3D wave pattern for $W_2(x,y,t)$ and the $\mathrm{Re}(W_2(x,y,t))$ showing **breather-like solitons**profiles for $p = q = a = \omega = 0.5$, $m_3 = \eta = \vartheta = 1$, $\varrho = 1.5$, and $y = 0$.

It is noticed from Figure 1-Figure 8 that both the solitons maintained their amplitudes, widths, and directions constant throughout the propagation time on $x - y$ coordinate. It is also noticed that, these parameters ϱ, η, m_3, p, q, a, ω, and ϑ, are regulating the wave propagation for these solutions. The cmKdV was investigated in the literature by utilizing other techniques, see (Ibrahim, Sabiu & Gambo, 2024; Shaikhova, Kutum & Myrzakulov, 2022; Muhammad, Sabi'u, Salahshour, & Rezazadeh, 2024). Moreover, the designed approach in this article has made a major contribution in obtaining numerous families of exact solutions of Eq. (1) which appear recent and recuperate the famous exact solutions obtained in (Ibrahim, Sabiu & Gambo, 2024; Shaikhova, Kutum & Myrzakulov, 2022; Muhammad, Sabi'u, Salahshour, & Rezazadeh, 2024) using the Ricatti, the tanh-coth, Kudryashov, sine-cosine and Sadar methods. Finally, all the obtained solutions reported in this research have been checked and authenticated with Maple 19 by substituting them into the original system, that is system Eq. (1). Thus, one advantage of the execution of the new extended direct algebra approach is that the present method is exceptionally reliable and gives unique solutions in comparison to other approaches, as well as the capacity to elaborate more abundant families of exact solutions comprising periodic, singular, dark, breather and bright solitons with varying parameters value used in the model. The techniques cited in (Ibrahim, Sabiu & Gambo, 2024; Shaikhova, Kutum & Myrzakulov, 2022; Muhammad, Sabi'u, Salahshour, & Rezazadeh, 2024) give bright, periodic, singular and bright soliton solutions, also the new extended direct algebra method gives singular, periodic, bright and dark soliton patterns. Furthermore, in this research, we have achieved 20 unique solutions for the cmKdV model. Whereas the techniques utilised in (Ibrahim, Sabiu & Gambo, 2024) yield only 12 solutions and (Shaikhova, Kutum & Myrzakulov, 2022) provide only 8 exact solutions for the whole three techniques utilised in the paper, this shows that the innovative extended direct algebra method is a reliable, sophisticated, accurate and effective method.

CONCLUSION

In this paper, we thoroughly studied the cmKdV model that has applications in water wave and plasma physics using the innovative direct algebra method. We obtained various exact solutions for the cmKdV model that give rise to different wave patterns like dark, multiple, singular, breather and bright wave patterns. The nature of these solutions is in the form of rational, finite exponential, hyperbolic and trigonometric functions. The study exhibits the intricate dynamics of propagating waves for parameter variations, implying various travelling wave behaviour. We applied different parameter values and provided unique graphical interpretations of some of the exact solutions, yielding a precious understanding of the governing model's evolution which has numerous applications in water waves, plasma physics, nonlinear optics and other contemporary sciences. Antecedent to this investigation, previous studies have not yielded solutions of this nature. The findings provide valuable insights into the complexities of nonlinear wave phenomena and reaffirm that the extended direct algebra method is a reliable and adaptable method for addressing complex NPDEs-related nonlinear problems. In the future, we will incorporate the concepts of modulation instability and Lie symmetry analysis for the considered model to explore more exciting features of this system.

Availability of data and materials

Data sharing does not apply to this article as no data sets were generated or analyzed during this study.

Conflict of interest

The authors declare no conflict of interest.

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