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Modelling the Impact of Long-Range Dependence and Heteroscedasticity on Economic Growth of Nigeria Using TAR-FIGARCH Model

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ABSTRACT
This research work is based on modelling the impact of long-range dependence, heteroscedasticity and regime switching in the economic growth of Nigeria using TAR-FIGARCH model on account to capture long-term memory persistence, changing variability, and identify different growth regime in the economic growth of Nigeria respectively. The data for this research work was collected from World Bank record spanning from (1960-2022) in dollar. ADF and KPSS Tests were used to test for stationary. Partial Autocorrelation Plot was used to estimate the model order. TAR Model with two regimes was fitted to analyze regime switching with quick and slow transitions and to segregate volatility into low and high states using delay parameters and variance weights. The total variance weights of the two regimes is (0.6259 + 0.3741 = 1), thus the model fits the time series. The estimated delay parameters of first and second regime are (55.3637 and 0.3173) respectively. The research work revealed that second regime is characterized by quick regime switching. Moreover the first regime has high volatility accounting for approximately (52.23%) of the total variance. Thus, this indicates economic instability, economic uncertainty and increased risk of economic downturns. The research work recommended that Central Bank should consider adjusting interest rates to stabilize the economic growth and Government should implement targeted fiscal policies to mitigate the impact of fluctuations. The future work can employ three regimes TAR Model to separate the economic growth into low, medium and high regime.

Keywords:
Long-Range Dependence,
Heteroscedasticity,
Regime Switching,
Economic Growth,
TAR,
FIGARCH.

INTRODUCTION

The issues with time series data that led to this research include heteroscedasticity, long memory or long range dependence, and regime switching. Heteroscedasticity is the presence of varying levels of volatility or dispersion in the error terms over time. The causes of heteroscedasticity are presence of outliers and presence of structural breaks. The consequences of the heteroscedasticity include biased standard errors of the estimated coefficients, which can affect parameter estimates, and make them less precise. Long memory or Long-Range Dependence is the serial correlation between the time series. The causes of long-range dependence are; changes in mean, changes in variance, changes in autocorrelations over time, external shocks, heavy-tailed distributions, and volatility clustering. The consequences of long-range dependence include; incorrect parameter estimates, reduced test power, false correlations, and incorrect model choice.

In concept of regime switching, many economic time series occasionally exhibit dramatic breaks in their behavior, associated with events such as financial crises (Jeanne and Masson, 2000; Cerra, 2005; Hamilton, 2005) or abrupt changes in government policy (Hamilton, 1988; Sims and Zha, 2004, Davig, 2004).

Threshold Autoregressive TAR model was developed by Tong (1983). It uses univariate time series data, stationary time series data, captures the mean of time series, andprovides values below and above the threshold; however; it does not capture variance and long memory or long-range dependence.

Fractional Integrated Generalized Autoregressive Conditional Heteroscedasticity FIGARCH Model was developed by Bollerslev and Mikkelsen (1996). It usesunivariate, stationary time series data, captures heteroscedasticity, variance, long memory and it does not provide values that are above and below the threshold. Jason (2017) used FIGARCH Model on Economic time series data, Hansen (2011) employed TAR Model on Economic time series and Chavez (2022) incorporate GARCH Model with TAR Model and formed hybrid momentum TAR-FIGARCH Model in order to address the weakness of TAR and GARCH model but the model cannot capture presence of long memory.

Studies were carried out to address the problems of long-range dependence, heteroscedasticity, and regime switching. For example Sadon et al. (2024) conducted a study on the impact of heteroscedasticity as a factor in forecasting future stock market trends using GARCH-LSTM hybrid model. Their findings demonstrated that hybrid GARCH-LSTM achieved lower MAE (7.961), RMSE (10.466), MAPE (0.516) and HMAE (0.005) values compared to a standalone LSTM model. Forecast accuracy also improved by 15% and 13% with the hybrid GARCH-LSTM relative to the single LSTM. Additionally, results indicated that hybrid GARCH-LSTM model fully leverages the heteroscedasticity component, which is not accounted for by GARCH model estimation alone, outperforming standalone GARCH models.

In attempt to model Chilean economic uncertainty,Chavez et al. (2022) carried out a study. In this study, an autoregressive moving average (ARMA) model with threshold generalized autoregressive conditional heteroscedasticity (TGARCH) innovations is considered to model Chilean economic uncertainty time series. The ARMA-TGARCH model is compared with the classical seasonal ARIMA and threshold AR ones. The results show that the ARMA-TGARCH model explains the regime changes in economic uncertainty better than the others, given that negative shocks are associated with statistically significant and quantitatively larger levels of volatility produced by the COVID-19 pandemic.

Bawa et al. (2023) conducted a study that focused on developing hybrid ARIMA-FIGARCH model for time series that has issues with long-range dependence, heteroscedasticity, volatility, and regime switching. This research aimed to create a novel ARIMA-FIGARCH hybrid model. The findings revealed that the proposed ARIMA-FIGARCH model demonstrated improved consistency, as performance metrics decreased and approached zero with increasing sample size.Wang et al. (2011) carried out a research on Hybrid Momentum TAR-GARCH Models for Short Term Load Forecasting. This research work aimed at investigating the volatility of the load time series for improving the performance of short term load forecasting. Results of the case study clearly validate the feasibility and effectiveness of the proposed models. Also, the model comparison for forecasting performance demonstrates that the Hybrid Momentum TAR-GARCH model with a generalized error distribution outperforms others based on several statistical criteria.

Hansen. (2011) conducted a research work on threshold autoregressive model in economics. This research work aims to document the influence of Howell Tong's TAR model in the fields of econometrics and economics. The research work revealed that at this review documents, econometrics and economics have been greatly influenced by the TAR model. Consequently, we owe a debt to Howell Tong for his visionary innovation.

Sadon et al. (2024) conducted a study on the impact of heteroscedasticity as a factor in forecasting future stock market trends using a GARCH-LSTM hybrid model. Their findings demonstrated that the hybrid GARCH-LSTM model achieved lower MAE (7.961), RMSE (10.466), MAPE (0.516), and HMAE (0.005) values compared to a standalone LSTM model. Forecast accuracy also improved by 15% and 13% with the hybrid GARCH-LSTM relative to the single LSTM. Additionally, results indicated that the hybrid GARCH-LSTM model fully leverages the heteroscedasticity component, which is not accounted for by GARCH model estimation alone, outperforming standalone GARCH models.

This study addresses limitations in the TAR and FIGARCH modelsby developing a non-linear hybrid time series model capable of capturing both models' distinct characteristics and limitations. The proposed model is applied to the economic growth time series of Nigeria. Key research questions include; (i) How do autocorrelation and heteroscedasticity influence the behavior and characteristics of economic time series data? (ii) Can the TAR Model effectively address autocorrelation and heteroscedasticity when combined with a FIGARCH Model? (iii) Does the proposed model reduce volatility persistence in the residuals of Nigeria's economic growth time series?

MATERIALS AND METHODS

Method of Data Collection:

The data used in this research is secondary data obtained from the macrotrends database, accessible at (https://www.macrotrends.net/globalmetrics/countries/NGA/nigeria/economicgrowth). This dataset records the annual economic growth of Nigeria from 1960 to 2022, providing a comprehensive historical overview for time series analysis.

Method of Data Analysis:

This section details the methods employed for the data analysis in this research. The analysis methods include: Time series visualization, stationary checking, model order estimation, TAR modelspecification, parameter estimation, variance weights and delay parameter estimation.

Time Series Visualization:

The time series time series visualization in this research was achieved using time series plot; R software was employed to visualize the behavior of the Nigeria's economic growth time series over an extended period.

Stationary Checking:

Stationarity of the economic growth time series was assessed using the Augmented Dickey-Fuller Test (ADF) and theKwiat-Kowski Phillips Smidth Shin (KPSS) test.

Augmented Dickey-Fuller Test:

The presence of unit roots in the time series was assessed using the Augmented Dickey-Fuller (ADF) developed by Said and Dickey-Fuller in 1984. The test is defined as follows:

$$\Delta Y_t = \alpha Y_{t-1} + \sum_{k=1}^{k} \theta_k \Delta Y_{t-k} + \mu_t \qquad (1)$$

where, k is the number of lags or autoregressive terms and $\sum_{k=1}^{k} \theta_k \Delta Y_{t-k}$ is the additional autoregressive terms.

The test has the following hypotheses:

H_0: the time series has unit root.

H_a: the time series non-unit root.

We are to reject the null hypothesis if P-value is less than the alpha value.

Kwiat-Kowski Smidth-Shin Test:

The presence of unit roots in the time series was assessed using the with the Kwiat-KowskiSmidth-Shin (1992) test, proposed by Kwiatkowski et al. in 1992. This test serves as the second approach to evaluate the unit roots of the economic growth time series data. The test is defined as follows:

$$KPSS_T = \frac{\sum_{t=1}^{T} (\sum_{i=1}^{t} \hat{\mu}_i)}{T^2 \sum_{i=(-T^{\frac{2}{9}})} \frac{1}{T-i} \sum_{t=i+1} \hat{\mu}_t \hat{\mu}_{t-i}} \qquad (2)$$

Where, the numerator is the cumulative residual function and the denominator is the long run variance of the error term.

The test has the following hypotheses:

H_0: the time series has no unit roots.

H_a: the time series has unit roots.

The null hypothesis is not rejected if the KPSS test statistic value obtained is less than the critical value at the alpha level of significance.

Estimation of Model Order:

This research employed the Partial Autocorrelation Function (PACF) Plot to identify the order of Autoregressive model to be included in the developed model. The partial autocorrelation function is defined as follows:

$$\rho_K = \frac{\sum_{t=k+1}^{n} (Y_t - \bar{Y})(Y_{t-k} - \bar{Y})}{\sum_{t=1}^{n} (Y_t - \bar{Y})^2} \qquad (4)$$

where, ρ_K is the autocorrelation at lag k, Y_t is the time series at time t, $\bar{Y}$ is the mean of the time series and n is the number of the observations.

TAR Model Component:

The Threshold Autoregressive (TAR) model, proposed by Tong in 1983, captures the mean behavior of a time series by separating the autoregressive model into two regimes: one for values that exceed the threshold and one for the values that do not. However, TAR model does not account for variance or the presence of long memory in the time series data. The model is defined as follows:

$$Y_t = \begin{cases} \varphi_{10} + \varphi_{11} Y_{t-1} + \cdots + \varphi_{1p} Y_{t-p} + \varepsilon_{1\tau} & \text{if } Y_{t-d} > \tau \\ \varphi_{20} + \varphi_{21} Y_{t-1} + \cdots + \varphi_{2p} Y_{t-p} + \varepsilon_{2\tau} & \text{if } Y_{t-d} \le \tau \end{cases} \qquad (5)$$

where, φis the Autoregressive parameters, Y_{t-d} is the threshold variable, τ is the threshold parameter and ε_t is the white noise, p is the order the model, t is the time and d is the delay lag.

Parameters Estimation of TAR Model:

$$Y_t = \phi_0 + \phi_1 y_{t-1} \qquad (6)$$

$$\phi_0 + \phi_1 y_{t-p} = Y_t \qquad (7)$$

Take summation both sides of equation (7)

$$\sum \phi_0 + \sum \phi_1 y_{t-1} = \sum Y_t \qquad (8)$$

Multiply y_{t-1} throughout equation (8)

$$\sum \phi_0 y_{t-1} + \sum \phi_1 y_{t-1} y_{t-1} = \sum y_t y_{t-1} \qquad (9)$$

$$\sum \phi_0 y_{t-1} + \sum \phi_1 y^2{}_{t-1} = \sum y_t y_{t-1} \qquad (10)$$

Divide equation (9) throughout by ny_{t-1}

$$\phi_0 + \frac{\sum \phi_1 y_{t-1}}{n} = \frac{\sum y_t}{n}$$

$$\phi_0 = \frac{\sum y_t - \phi_1 \sum y_{t-1}}{n} \qquad (11)$$

Now put equation (11) into equation (10)

$$\left(\frac{\sum y_t - \phi_1 \sum y_{t-1}}{n} \right) \sum y_{t-1} + \sum \phi_1 y^2{}_{t-1} = \sum y_t y_{t-1}$$

$$\left(\frac{\sum y_t y_{t-1} - \phi_1 \sum y^2{}_{t-1}}{n} \right) +$$

$$\sum \phi_1 y^2{}_{t-1} = \sum y_t y_{t-1}$$

$$\sum y_t y_{t-1} - \phi_1 \sum y^2{}_{t-1} + n \sum \phi_1 y^2{}_{t-1} = n$$

$$\sum y_t y_{t-1}$$

$$n \sum \phi_1 y^2{}_{t-1} - \phi_1 \sum y^2{}_{t-1} = n \sum y_t y_{t-1} - \sum y_t y_{t-1}$$

$$\phi_1 \sum y^2{}_{t-1}(n-1) = \sum y_t y_{t-1}(n-1)$$

$$\phi_1 = \frac{\sum y_t y_{t-1}}{\sum y^2{}_{t-1}}$$

$$\phi_2 = \frac{\sum y_t y_{t-1}}{\sum y^2{}_{t-1}}$$

Variance Weights Estimation:
Variance weights are employed in this research to explain the proportion of variance accounted for in each regime. These weights are used to assess the goodness of fit of the model to the time series. Notably, the normalized sum of variance weights is always equal to one.

Delay Parameter:
This research estimated delay parameter to measure the number of time periods required for the system to full transition from one regime to another.

RESULTS AND DISCUSSION

Behavior Checking of the Economic Growth Time Series

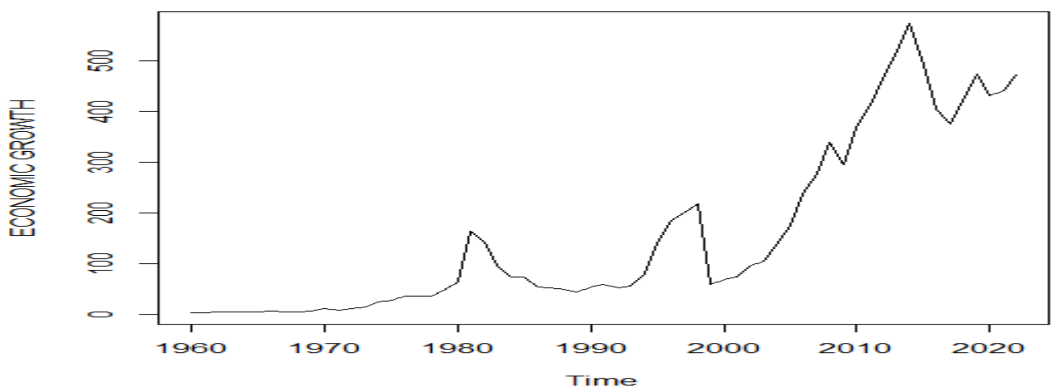

Figure 1: Behavioral Pattern of Economic Growth Time Series

From the result obtained in Figure 1, it has been observed that the long term behavior of the economic growth in Nigeria is trending upward.

Stationary Checking of the Economic Growth Time Series using ADF Test:
H_0: the time has unit root (non-stationary).
H_a: the time series has non-unit root (stationary).

Table 1: ADF Test Results

Dickey-Fuller	Lag Value	Probability Value
-1.8944	3	0.6177

From the above result obtained in table 1 it has been observed that the ADF P-value is 0.6177. This is greater than the alpha value 0.05. Thus, we fail to reject the null hypothesis and conclude that the economic time series is not stationary. Thus, we have to difference the data to make it stationary. But before we difference the data is good to for another test of stationary using KPSS Test which is an opposite of ADF test.

Stationary Checking of the Economic Growth Time Series using KPSS Test:
H_0: the time has non-unit root (stationary).
H_a: the time series has unit root (not stationary).

Table 2: KPSS Test Results

KPSS Level	Lag Value	Probability Value
1.3269	3	0.01

From the result obtained in Table 2, it is observed that the probability value at lag (3) is 0.01 which is smaller than the critical alpha value 0.05. Therefore, we reject null hypothesis and conclude that the economic growth time series is not stationary. As a result, we need to difference the time series to achieve stationarity.

Behavioral Pattern of the Differenced Economic Growth Time Series:

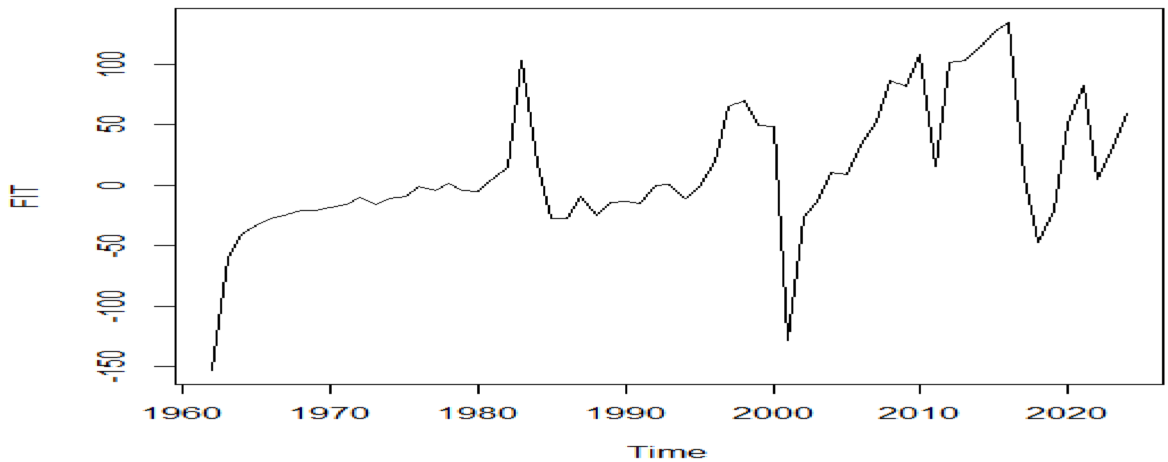

Figure 2: Behavioral Patterns of the Differenced Economc Growth Time Series

From the result obtained in Figure 2, it is observed that the behavior of the time series resembles random walk. This indicates that the time series is now stationary, meaning that the mean of the time series is constant. However, since this conclusion is based on visualization, we should conduct additional tests of stationary to confirm the result obtained from the time series plot.

Partial Autocorrelation Plot of the Differenced Economic Growth Time Series:

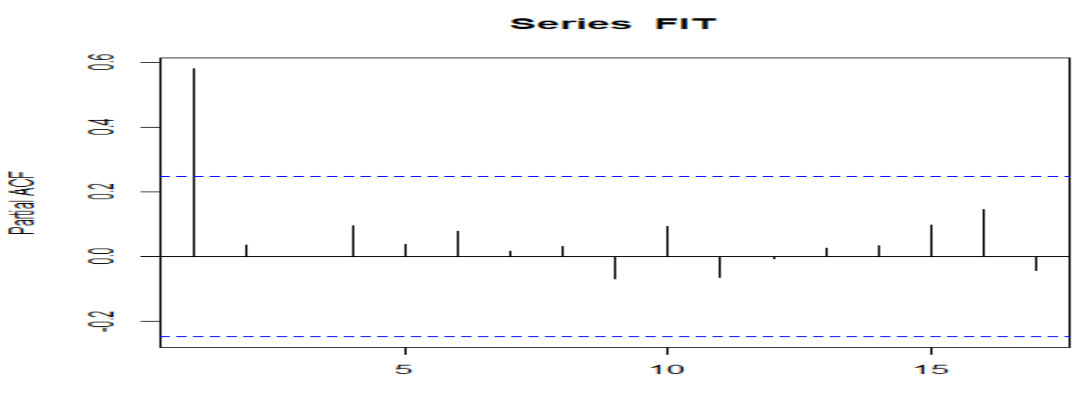

Figure 3: Partial Autocorrelation Plot

From the result obtained in Figure 3, it is observed that a significant spike occurs at lag 1, indicating that AR (1) term is significant.Therefore, we will fit a TAR (2, (1, 1)) model.

Stationary Checking of the Differenced Economic Growth Time Series using ADF Test:

H_0**:** the time has unit root (non-stationary).

H_a: the time series has non-unit root (stationary).

Table 3. ADF Test results of the Differenced Time Series

Value of test-statistic is: -3.5189 4.2813 6.2441			
Critical values for test statistics:			
Values	1PCT	5PCT	10PCT
Tau3	-4.04	-3.45	-3.15

From the result obtained in Table 3 it is observed that the ADF test statistic is -3. 5189, which is less than the critical value at 5% level of significance. Therefore, wereject the null hypothesis and conclude that the economic time series is stationary.

Stationary Checking of the Economic Growth Time Series using KPSS Test:

H_0: the time has non-unit root (stationary).
H_a: the time series has unit root (not stationary).

Table 4. KPSS Test Results of the Differenced Time Series

Value of test-statistic is: 0.0541			
Critical value for a significance level of:			
10PCT	5PCT	2.5 PCT	1PCT
0.119	0.146	0.176	0.216

From the result obtained in Table 4, it is observed that the KPSS test statistic is 0.0541, which is smaller than the critical value at 5% level of significance. Therefore, we reject the null hypothesis and conclude that the economic growth time series is stationary.

TAR (2(1, 1)) using Lagged Threshold Variable and Standard Deviation Threshold:

Table 5 Autoregressive Coefficient of the First Regime

Lag	Estimate	Std.Error Limit	Limit Inferior	Limit Superior
Lag0	10.38584504	6.52357067	-1.5736167	21.93729929
Lag1	-0.05489835	0.02471486	-0.1019618	-0.01077996

Table 6 Autoregressive Coefficient of the Second Regime

Lag	Estimate	Std.Error Limit	Limit inferior	Limit superior
Lag0	0.9831319	4.004538120	-6.4297363	9.866362
Lag1	1.0000103	0.002013026	0.9955369	1.003728

From the results obtained in Table 6 and Equation 1, it is observed that the first regime is characterized by a strong positive effect of the current value (10.39) and weak negative effect of the previous value (-0.05). The second regime is characterized by positive effect of the current value (0.98) and strong positive effect of the previous value (1.00). The threshold variable y_{t-25} determines which regime is active at each time point.

Table 7: The Estimated Variance Weights

Regime	Estimate	Std.Error Limit	Limit Inferior	Limit Superior
Regime 1	52.2304086	4.86567295	44.7053702	63.4854238
Regime 2	0.3122586	0.03057558	0.2630811	0.3685963

Based on the results presented in Table 7, it is observed that Regime One accounts for approximately 52.23% of the total variance in economic growth. This high variance weight indicates that Regime One is characterized by high volatility or substantial fluctuations. Conversely, Regime Two accounts for approximately 31.23% of the total variance in economic growth, with its low variance weight suggesting that it is characterized by low volatility or minor fluctuations. The normalized variance weights for each regime are 0.63 and 0.37, respectively, and together they sum to one. Thus, the model demonstrates a good fit to the data.

Table 8: Estimated Theta (ϴ) Values

Regime 1	Regime 2
-1.70217784	-0.75320931

Based on the results presented in Table 8, the threshold value for switching from Regime One to Regime Two is observed to be -1.70. This indicates that when the time series exceeds -1.70, it transitions to Regime Two. Additionally, the threshold value for switching from Regime Two back to Regime One is -0.75, meaning that when the time series falls below -0.75, it returns to Regime One.

Table 9: The Estimated Delay Parameters:

Regime 1	Regime 2
55.36372	0.3173403
51.27116	0.3501259

From the results presented in Table 9, it is observed that the first regime has high values for the estimated delay parameters, indicating that the effect of the threshold variable on the time series is delayed by a significant number of periods (approximately 51-55 periods). This finding suggests that, in the first regime, the economy requires a substantial amount of time to respond to changes in the threshold variable. In contrast, the second regime has low values for the estimated delay parameters, indicating that the effect of the threshold variable on the time series is nearly immediate or occurs with a very short delay (less than one period).

CONCLUSION

Based on the analysis, the TAR model was found to fit the time series data effectively, as indicated by the normalized variance weights summing to one. The analysis also revealed that volatility is higher in the first regime and lower in the second, with each regime exhibiting distinct response patterns to the current and previous values. Specifically, the first regime shows a strong positive influence from the current value and a weak negative influence from the previous value, while the second regime shows a weak positive influence from both the current and previous values. These findings suggest that economic responses differ significantly between regimes, with implications for understanding and forecasting time series behavior under varying conditions.

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