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A Modified Sequential Probability Ratio Test for Truncated Life Tests Using Log-Normal and Inverse Exponential Rayleigh Distribution

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ABSTRACT

This research developed a Modified Sequential Probability Ratio Test (MSPRT) for Log Normal and Inverse Exponential Rayleigh Distributions based on truncated life testing. MSPRT is a mechanism designed to minimise the average sample size necessary for conducting statistical hypothesis tests at predetermined significance levels (α) and power (1-β) across various maximum sample sizes (N), indicating that there exists a definitive termination threshold for the minimum number of items required to implement a sampling plan. The study findings compared the maximum sample size necessary for the test at a 5% significance level across various areas. The study concluded that as the duration of test termination increases, the maximum required sample size diminishes, while the values in the graphical representation rise more steeply with improved quality. Consequently, the proposed plan was deemed optimal for selecting the appropriate sample size. In comparison to the two tests, it is evident that the current ASN values at levels 10, 15, and 25 exceed the proposed ASN values of 9, 11, and 18, respectively. This indicates that at a specific ASN level, the proposed plan can be accepted and implemented to reduce inspection time and simultaneously save costs, thereby representing the optimal plan in relation to the existing sequential probability ratio test.

Keywords:
Sequential sampling plan, Acceptance sampling, Truncated life test, exponential Rayleigh distribution, Producer and consumer risks

INTRODUCTION

Statistical quality control involves the implementation of various methods for monitoring and maintenance of the quality of products as well as services. Quality is now not only an optionor aim of companies but a necessity for businesses in a global market (Balakrishnan*et al.*, 2007). Acceptance sampling plans play a very important role in the statistical quality control, especially in the lot production process to decide whether to reject or accept the lot (Al-Nasser & Al-Omari, 2013). The decision on the quality of all entire items in each lot depends on drawing a random sample of size *n* from a selected lot; after that, within a specific timeframe, testing procedure is initiated to discover the number of failure or defective items included in the sample before the pre-indicated time is terminated (Al-Nasser *et al.*, 2016; Al-Omari *et al.*, 2016).

Sequential probability acceptance sampling is an important quality control tool in terms of making decisions about a particular lot. As companies all over the world are taking improvement quality of their product to be uttermost, it is not surprise that study on sequential acceptance sampling is paramount. Acceptance sampling plans are important tools widely used for promoting product quality in the industries. It is an inspection procedure concerned with accepting or rejecting a given lot of large size of products on the basis of its quality after inspection of a sample taken from the lot (Zoramawa*et al.*,2018).Sequential sampling is an extension of the double-sampling and multiple-sampling concept. However, sequential sampling takes a sequence of samples from the lot and allows the number of samples to be determined entirely by the results of the sampling process. In practice, sequential sampling can theoretically continue indefinitely, until the lot is inspected 100%. Usually, a sequential sampling plan is truncated after the number of inspections is equal to three times the number that would have been inspected using a corresponding single sampling plan. This approach allows one to draw conclusions during the data collection, and a final conclusion can possibly be reached at a much earlier stage as is the case in Classical Hypothesis Testing. Since the 18th century there has been a growing interest in statistical hypothesis testing. In literatures, several theories have been already proposed to extend the applicability of the mathematical background or to optimize calculation. MSPRTs are the firm requirement to specify the outcome variable and test statistic prior to the start of the experiment. Of course, in principle the same requirement applies to fixed design experiments, but failure to ensure that these quantities are clearly identified a priori could lead to additional opportunities for *p* hacking and other unethical practices in sequential designs. For instance, researchers might apply MSPRTs to several outcome variables simultaneously, which would negatively affect the control of Type I errors. In addition, the conduct of MSPRTs requires that investigators perform statistical analyses after the acquisition of each participant's data, which in some settings may not be feasible. However, for studies in which a high threshold for significance is desired, this technique may offer researchers a method of testing hypotheses while maintaining required sample sizes at a manageable level. The modified sequential probability ratio test thus encourages the design of tests that will lead to Bayes factors (or likelihood ratios) that differ substantially from and they do so with smaller sample sizes than are required in fixed design tests.

This study was motivated by Mayssa and Ali (2021) where SPRT was proposed for Parameter estimation of inverse exponential Rayleigh distribution and Log-Normal Distribution based on classical methods in which the statistical properties such as probability density, cumulative, survival, hazard, quantile, and moment functions were computed by the study. Theclassical SPRT has a limitation,which is, hasty decision in terms of rejecting the entire lot due to a faulty inspection and this could potentially damage the relationship that exist between the consumer and the producer. As a result, this research introduces the Modified SPRT in other to handle such cases.

The aim of this research was to develop sequential probability ratio test using modified Log-Normal and Inverse Exponential Rayleigh Distributions, specifically, this was achieved through obtaining the AQL and LTPD from the modified log-normal distributionand Inverse exponential Rayleigh distributions, determining the minimum itemsfor inspection that ensure the specified mean lifetime for the plan, construction of the diagrammatic representation for the plan and finally determine the acceptance sampling number (ASN) for the modified SPRT plan. Many researchers have developed Acceptance sampling plans using sequential, single and double method of sampling plans to ensure product quality standard, every industry must develop suitable mechanism by adopting suitable statistical quality control techniques to emphasize the acceptability of a lot based on a random sample selected from the product.

Zoramawa and Charanchi (2021), used Sequential probability sampling analysis to treat the sample size obtained from either single or double sampling plans. Precisely the research considers and compared the minimum sample obtained from Bur Type XII distribution. Estimations of minimum sample, acceptance and rejection numbers obtained were analyzed and presented to explain the usefulness of sequential plans in relation to single and double sampling plan. Average Sample Number (ASN) obtained indicated the hypothesis at various risks' levels was accepted indicating there is a time limit to terminate the sampling. Sequential probability sampling (SPS) plays a vital role at any sampling plan obtained using Bur Type XII distribution and saves inspection time which was among the major concern of both producers and consumers in the manufacturing industries.

However, Mayssa and Ali (2021) proposed SPRT for Parameter estimation of inverse exponential Rayleigh distribution and Log-Normal Distribution based on classical methods in which the statistical properties such as probability density, cumulative, survival, hazard, quantile, and moment functions were computed by the study.

Harsh and Mahendra (2021) introduced single acceptance sampling inspection plan (SASIP) for transmuted Rayleigh (TR) distribution when the lifetime experiment is truncated at a prefixed time. Establish the proposed plan for different choices of confidence level, acceptance number and ratio of true mean lifetime to specified mean lifetime. Minimum sample size necessary to ensure a certain specified lifetime is obtained. Operating characteristic (OC) values and producer's risk of proposed plan are presented.

Sandipan, *et al.,* (2021) described a modified sequential probability ratio test which used to reduce the average sample size required to perform statistical hypothesis tests at specified levels of significance and power that provided z tests, t tests, and tests of binomial success probabilities and compare the sample sizes required in fixed design tests conducted at 5% significance levels to the average sample sizes required in sequential tests conducted at 0.5% significance levels, which found that both the two sample sizes are approximately equal.

The primary objective of SPRT, as indicated in the aforementioned articles, is to determine the least number of items to be inspected that adequately represent the lifespan of the product in question. This research presents a modified Sequential Probability Ratio Test (SPRT) that employs log-normal and inverse exponential Rayleigh distributions to determine the best sample size required for manufacturing industries to minimize inspection time and costs.

MATERIALS AND METHODS

The SPRT is one of the most widely known sequential testing procedures (Wald, 1945; Lai, 2001, 2004, 2008; Bartroff *et al.*, (2008); Bar and Tabrikian, 2018). This test is based on comparing the likelihood ratio between a simple (i.e., point or precise) null hypothesis and a simple alternative hypothesis, and stopping data collection as soon as the likelihood ratio strongly supports one of the two.

Limitation of the SPRT is that the sample size required to complete a test cannot be determined prior to the start of data collection. In nearly all experimental settings, resources available for testing participants are limited and in observational studies the amount of the data that can be collected from a population is finite. This feature of the SPRT thus complicates the practical design of tests and is resolved by the MSPRT. An earlier modification of the SPRT, known as the truncated SPRT, was proposed by Anderson (1960) to address this difficulty. However, this modification generally provides less statistical power than our proposed MSPRT. in Siegmund (1985) indicate that for the alternative effect size that provides 90% power in a fixed design test, the truncated SPRT's power is only 84%. By comparison, the MSPRT provides between 88–89% power at the same alternative.

To illustrate this procedure, suppose that independent data values are collected sequentially, denoted by these values by $x_1, x_2, \ldots$. Suppose further that the null hypothesis implies that the probability density function describing a single data value x_i is $f(x_i \mid \theta_0)$, and that the alternative hypothesis implies that the probability density function is $f(x_i \mid \theta_1)$. Then the likelihood ratio in favor of the alternative hypothesis based on the first n observations is defined as

$$L(\theta_0, \theta_1; n) = \prod_{i=1}^{n} \frac{f(x_0, \theta_1)}{f(x_1, \theta_0)} \qquad (1)$$

To simplify notation, we denote $L(\theta_0, \theta_1; n)$ by L_n.

Heuristically, the MSPRT keeps track of the likelihood ratio L_n as data accumulate, and stops the experiment as soon as the probability assigned to the data under one hypothesis significantly exceeds the probability assigned to the data by the other hypothesis.

More formally, the MSPRT proceeds by comparing L_n, $n = 1, 2, \ldots n$ to some constants A and B, where $A > B > 0$, as data from individual study participants are collected. The procedure stops when $L_n \geq A$ or $L_n \leq B$, or equivalently when L_n exits the interval (B, A) for the first time. The quantities A and B are defined as:

$$A = \frac{1-\beta}{\alpha} \text{ and } B = \frac{\beta}{1-\alpha} \qquad (2)$$

If $L_n \geq A$, the null hypothesis is rejected; if $L_n \leq B$, the alternative hypothesis is rejected. An important property of the SPRT is that it requires, on average, fewer participants to achieve its specified Type I and Type II error probabilities than any other test whose error probabilities are smaller than or the same as these (Wald and Wolfowitz, 1948).

MSPRT for Log Normal Distribution

In probability theory, a lognormal distribution is a continuous probability distribution of a random variable whose logarithm is normally distributed. Thus, if the random variable X is lognormal distributed, then Y=In(X) has a normal distribution, the mean of lognormal distribution and the variance are given respectively as

$$\text{mean} = \exp\left(\mu + \frac{\sigma^2}{2}\right) \text{ and variance}$$

$$= \left[\exp(\sigma^2) - 1\right]\exp(2\mu + \sigma^2) \qquad (3)$$

The probability density function (pdf) and cumulative distribution function for Log-Normal Distribution arerespectively given as:

$$PDF = \frac{1}{x\sigma^2\sqrt{2\pi}} \exp^{\left(-\frac{(\ln x - \mu)^2}{2\sigma^2}\right)} \qquad (4)$$

$$CDF = \frac{1}{2}\left[1 + erf\left(\frac{In\, x - \mu}{\sigma\sqrt{2}}\right)\right] \qquad (5)$$

The SPRT computationsfor the two hypothesis using the normal distribution is given by

$$p_{1n} = \frac{1}{(2\pi)^{\frac{n}{2\delta_1^2}}} e^{-\frac{1}{2\delta_1^2}\sum_{i=1}^{n}(x_i - \mu)^2} \qquad (6)$$

$$p_{2n} = \frac{1}{(2\pi)^{\frac{n}{2\delta_2^2}}} e^{-\frac{1}{2\delta_2^2}\sum_{i=1}^{n}(x_i - \mu)^2} \qquad (7)$$

If $\mu = \mu_1$ the probability ratio $\frac{p_{2n}}{p_{1n}}$ computed at each stage of inspection and additional observation is taken as long as:

$$B < \frac{p_{2n}}{p_{1n}} = \frac{e^{-\frac{1}{2\delta_2^2}\sum_{i=1}^{n}(x_i-\mu)^2}}{e^{-\frac{1}{2\delta_1^2}\sum_{i=1}^{n}(x_i-\mu)^2}} < A$$

The inspection is terminated with acceptance of the lot if

$$\frac{e^{-\frac{1}{2\delta_2^2}\sum_{i=1}^{n}(x_i-\mu)^2}}{e^{-\frac{1}{2\delta_1^2}\sum_{i=1}^{n}(x_i-\mu)^2}} \le B$$

Inspection is terminated with rejection of lot if

$$\frac{e^{-\frac{1}{2\delta_2^2}\sum_{i=1}^{n}(x_i-\mu)^2}}{e^{-\frac{1}{2\delta_1^2}\sum_{i=1}^{n}(x_i-\mu)^2}} > A$$

Where the approximate values of A and B are given by: $\frac{1-\beta}{\alpha}$ and $\frac{\beta}{1-\alpha}$ respectively by taking the inequalities of equation

$$\log\frac{\beta}{1-\alpha} < \frac{\mu_2-\mu_1}{\delta^2}\sum_{1=i}^{n}x_i + \frac{n}{2\delta^2}(\mu_1{}^2-\mu_2{}^2) < \log\frac{1-\beta}{\alpha}$$

For each n number of sample compute, the acceptance number

$$a_n = \frac{\mu_2-\mu_1}{\delta^2}\log\frac{\beta}{1-\alpha} + n\frac{\mu_2+\mu_1}{2}$$

and rejection number

$$r_n = \frac{\mu_2-\mu_1}{\delta^2}\log\frac{1-\beta}{\alpha} + n\frac{\mu_2+\mu_1}{2}$$

Then the Probability Density Function for Lognormal distribution is

$$f(x) = \frac{1}{x\delta\sqrt{2\pi}}e^{-\left(\frac{\ln x_i-\mu}{2\delta}\right)^2} \quad (8)$$

$$P_{1n} = \frac{1}{(2\pi)^{\frac{n}{2}}\delta_1^2}e^{-}\frac{1}{2\delta_1{}^2}\sum_{i=1}^{n}(\ln x_i-\mu)^2 \quad (9)$$

$$P_{2n} = \frac{1}{(2\pi)^{\frac{n}{2}}\delta_2^2}e^{-}\frac{1}{2\delta_2{}^2}\sum_{i=1}^{n}(\ln x_i-\mu)^2 \quad (10)$$

The probability ratio $\frac{P_{2n}}{P_{1n}}$ is computed at each stage of experiment, additional observation is taking as long as:

$$\frac{\beta}{1-\alpha} < \frac{P_{2n}}{P_{1n}} = \frac{\frac{1}{\delta_2{}^n}e^{-}\frac{1}{2\delta_2{}^2}\sum_{i=1}^{n}(\ln x_i-\mu)^2}{\frac{1}{\delta_1{}^n}e^{-}\frac{1}{2\delta_1{}^2}\sum_{i=1}^{n}(\ln x_i-\mu)^2} < \frac{1-\beta}{\alpha}$$

The SPRT is classified as satisfactory if

$$\sum_{i=1}^{n}(\ln x_i-\mu)^2 \le \frac{2\log\frac{\beta}{1-\alpha} + n\log\frac{\delta_2^2}{\delta_1^2}}{\frac{1}{\delta_1^2}+\frac{1}{\delta_2^2}}$$

The SPRT is classified as substandard if

$$\frac{\frac{1}{\delta_2{}^n}e^{-}\frac{1}{2\delta_2{}^2}\sum_{i=1}^{n}(\ln x_i-\mu)^2}{\frac{1}{\delta_1{}^n}e^{-}\frac{1}{2\delta_1{}^2}\sum_{i=1}^{n}(\ln x_i-\mu)^2} \ge \frac{\beta}{1-\alpha}$$

$$\frac{2\log\frac{\beta}{1-\alpha} + n\log\frac{\delta_2^2}{\delta_1^2}}{\frac{1}{\delta_1^2}+\frac{1}{\delta_2^2}} < \sum_{i=1}^{n}(\ln x_i-\mu)^2 < \frac{2\log\frac{1-\beta}{\alpha} + n\log\frac{\delta_2^2}{\delta_1^2}}{\frac{1}{\delta_1^2}+\frac{1}{\delta_2^2}}$$

$$\sum_{i=1}^{n}(\ln x_i-\mu)^2 \le \frac{2\log\frac{\beta}{1-\alpha} + n\log\frac{\delta_2^2}{\delta_1^2}}{\frac{1}{\delta_1^2}+\frac{1}{\delta_2^2}}$$

$$\sum_{i=1}^{n}(\ln x_i-\mu)^2 \ge \frac{2\log\frac{1-\beta}{\alpha} + n\log\frac{\delta_2^2}{\delta_1^2}}{\frac{1}{\delta_1^2}+\frac{1}{\delta_2^2}}$$

The Acceptance number will be computed as

$$a_n = \frac{2\log\frac{\beta}{1-\alpha}}{\frac{1}{\delta_1^2}+\frac{1}{\delta_2^2}} + \frac{n\log\frac{\delta_2^2}{\delta_1^2}}{\frac{1}{\delta_1^2}+\frac{1}{\delta_2^2}} \quad (11)$$

And rejection number will be computed as

$$r_n = \frac{2\log\frac{1-\beta}{\alpha}}{\frac{1}{\delta_1^2}+\frac{1}{\delta_2^2}} + \frac{n\log\frac{\delta_2^2}{\delta_1^2}}{\frac{1}{\delta_1^2}+\frac{1}{\delta_2^2}} \quad (12)$$

Inspection continues as long as

$$a_n \leq \sum_{i=1}^{n}\left(\ln x_i - \mu\right)^2 < r_n \tag{13}$$

Accept if

$$\sum_{i=1}^{n}\left(\ln x_i - \mu\right)^2 \leq a_n$$

Reject if

$$\sum_{i=1}^{n}\left(\ln x_i - \mu\right)^2 > a$$

The MSPRT for Inverse Exponential Rayleigh Distributionis given as

$$f\left(x,\theta,\gamma\right) = \gamma + \theta x e^{-\left(\gamma x + \frac{\gamma}{2x}\right)} \tag{14}$$

The likelihood function will be expressed as

$$p_{1n} = r_1 + \theta_1 x_1 e^{-}\left(r_1 x_1 + \frac{\theta_1}{2x_i^2}\right) \tag{15}$$

$$p_{2n} = r_2 + \theta_2 x_i e^{-}\left(r_2 x_i + \frac{\theta}{2x_i^2}\right) \tag{16}$$

$$\ln\prod_{1=i}^{n}\left(\frac{p_{2n}}{p_{1n}}\right) = \ln\prod_{1=i}^{n}\left(\frac{\gamma_2+\theta_2 x_1}{\gamma_1+\theta_1 x_1}\right) e^{-\left(\frac{\gamma_2 x_1 + \frac{\theta_2}{2x_1^2}}{\gamma_1 x_i + \frac{\theta_1}{2x_i^2}}\right)}$$

$$\ln\prod_{1=i}^{n}\left(\frac{p_{2n}}{p_{1n}}\right)$$

$$= \ln\left(\frac{\gamma_2+\theta_2 x_1}{\gamma_1+\theta_1 x_1}\right)^n e^{-\left(\gamma_2\sum x_i + \frac{\theta_1}{2\sum x_i^2} - \left(\gamma_1\sum x_1 + \frac{\theta_1}{2\sum x_i^2}\right)\right)}$$

$$\ln\prod_{1=i}^{n}\left(\frac{p_{2n}}{p_{1n}}\right) = n\ln\left(\frac{\gamma_2+\theta_2\sum x_i}{\gamma_1+\theta_1\sum x_1}\right)$$

$$-\frac{1}{\sum x_i}\left(\gamma_2-\gamma_1\right) - \frac{1}{2\sum x_1^2}\left(\theta_2-\theta_1\right)$$

$$\ln\prod_{1=i}^{n}\left(\frac{p_{2n}}{p_{1n}}\right) = n\ln\left(\frac{\gamma_2}{\gamma_1}+\frac{\theta_2}{\theta_1}\right)$$

$$-\frac{1}{\sum x_i}\left(\gamma_2-\gamma_1-\frac{1}{2}\left(\theta_2-\theta_1\right)\right) \tag{17}$$

$$B < n\ln\left(\frac{\gamma_2}{\gamma_1}+\frac{\theta_2}{\theta_1}\right) - \frac{1}{\sum x_i}\left(\gamma_2-\gamma_1-\frac{1}{2}\left(\theta_2-\theta_0\right)\right) < A$$

where A and B are SPRT Functions

$$\therefore\left(\gamma_2-\gamma_1\right) - \frac{1}{2}\left(\theta_2+\theta_1\right)\left(\frac{\theta}{1-A}\right) + n\ln\left(\frac{\gamma_2}{\gamma_1}+\frac{\theta_2}{\theta_1}\right)$$

$$< \frac{1}{\sum x_i} < \left(\gamma_2-\gamma_1\right)$$

$$-\frac{1}{2}\left(\theta_2+\theta_1\right)\left(\frac{1-\theta}{A}\right) < n\ln\left(\frac{\gamma_2}{\gamma_1}+\frac{\theta_2}{\theta_1}\right)$$

The MSPRT for two-sided tests is accomplished by simultaneously running two one-sided tests of size $\alpha/2$. Before reaching the maximum sample size, the test terminates by (*a*) rejecting H_0 when either of the tests reject H_0, or (*b*) by not rejecting H_0 if both the tests reject H_1. If the test continues to the maximum sample size, then a common termination threshold, γ, is determined so as to maintain the desired Type I error probability of the test The design parameter γ is chosen to be as small as possible while still maintaining the specified size of the test, α. If $L_N \geq \gamma$ for either of the tests, the null hypothesis is rejected. Otherwise, the test rejects the alternative hypothesis.

RESULTS AND DISCUSSION

The Modified sequential probability ratio test is presented by computing the two regions then decision would be taken based on the regions by the use of the average sample number conditions for either accepting or rejection. The average sample number was computed using upper and lower regions at different category. The regions were plotted based on the various risk value β as shown in table 1 below.

Table 1: The values for β, maximum number of g's (n) with constant $\alpha = 0.05$, $p_0 = AQL = 0.05$ and $p_1 = LTPD = 0.2$

	Power		No. of exp.
s/n	$1-\beta$	β	Max n
1	0.80	0.20	**10**
2	0.85	0.15	**15**
3	0.96	0.04	**25**

Table 1 shows the summary of the result obtained from the proposed plan. The outcome shows that at $\beta = 0.20$, various sample sizes were used to compute the probability of acceptance where 10 was the maximum number of samples at this stage. Similarly, that at $\beta = 0.15$, the maximum number used to compute the probability of acceptance was found to be 15. at $\beta = 0.04$, 25 was the highest. The Analysis conducted regarding Modified SPRT was based on these three outcomes as shown in the summary of the result table.

Table 2: MSPRT For Log Normal Distribution (LND) with computation at power $P^* = 1 - \beta = 0.85$, Lower prop. (AQL) $= p_0 = 0.05$, Higher prop.(LTPD) $= p_1 = 0.20$, max $n = 15$.

Trial	Lower Limit (d_1)	Higher Limit (d_2)	Accept	Reject
N	$d_1 = -h_1 + s_n$	$d_2 = h_2 + s_n$	$0 \leq c < d_1$	$d_2 < r < n$
1	-1.1	1.9	*	*
2	-1	2	*	*
3	-0.9	2.1	*	71.6%
4	-0.7	2.3	*	56.5%
5	-0.6	2.4	*	47.4%
6	-0.5	2.5	*	41.3%
7	-0.4	2.6	*	37%
8	-0.3	2.7	*	33.8%
9	-0.2	2.8	*	31.2%
10	-0.1	2.9	*	29.2%
11	0	3	0.3%	27.6%
12	0.1	3.1	1.2%	26.2%
13	0.2	3.3	1.9%	25%
14	0.4	3.4	2.6%	24%
15	0.5	3.5	3.1%	23.2%

Table 2 presented the computations of LND at second stage for modified sequential sampling plan and the maximum number of items to be inspected is tabulated whereas the probability of acceptance of its respective sample size are determined. The two lines plots indicated the acceptance (lower limit) and rejection (higher limit) lines, these lines were computed and plotted on the x and y axis. The region in-between these lines are called no decision region as such continues inspection is necessary which shows that from item 1 to 10 it is recommended to continue sampling, one can decide to accept such lot when the 11thitems is inspected and no any defective item is found.

Table 3: Average Sample Number (ASN) result for β, maximum number of g's (n)with constant α, p_0 and p_1

h_1	h_2	ASN	$p = 0$	$p = 1$
1.0000	1.7794	ASN1	9.067	2.0019
1.1486	1.8183	ASN2	10.756	2.0437
2.0329	1.3964	ASN3	18.433	2.1315

Table 3 presented the general summary of the MSPRT limits with its corresponding average sample number ASN. These limits were used to plot the three regions in order to make decision of either to accept, reject the lot or continue sampling.The MSPRT regions and plots at power $P^* = 1 - \beta = 0.80,\ \ 0.85$, 0.96, lower proportion (AQL) $p_0 = 0.05$, higher prop.(LTPD) $p_1 = 0.20$, max $n = 10{,}15\ and\ 25$

The slope s for the hall computation across the work was given by

$$s = \frac{g_2}{g_1 + g_2} = \frac{\log_{10}\left(\frac{1-p_0}{1-p_1}\right)}{k} = {0.0746}/{0.6767} = 0.1102$$

The MSPRT limits for the first category

$d_1 = -h_1 + Sn = -1.0000 \quad + 0.1102\text{n}i = 1 \dots n$

$d_2 = h_2 + Sn = 1.7794 + 0.1102\text{n}$

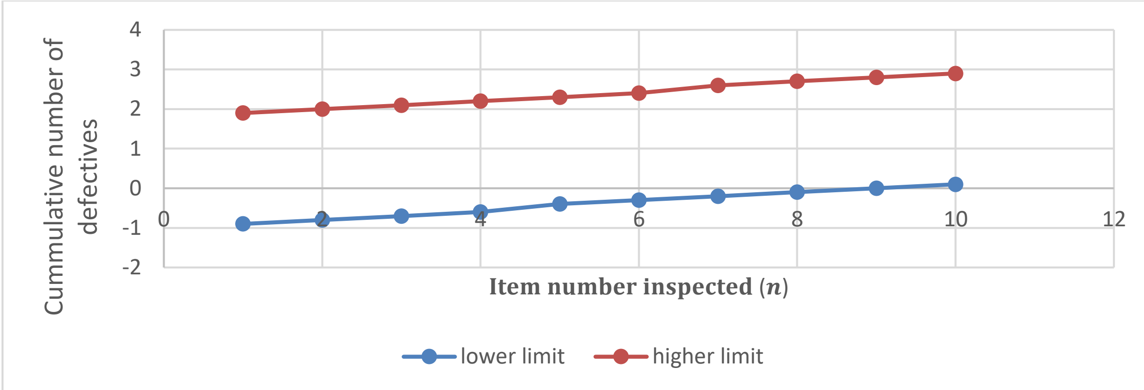

Figure 1: MSPRT plot at power $P^* = 1 - \beta = 0.80$, Lower prop. (AQL) $p_0 = 0.05$, Higher proportion.(LTPD) $p_1 = 0.20$, max $n = 10$.

Figure 1 presented the diagrammatic result of MSPRT with the upper limit d_2 and the lower limit d_1. The computed average sample number ASN at $p = 0$ was 9 which in comparison was less than the assigned number 10, that is $n \geq \frac{h_1}{s}$. This shows that the lot was accepted before reaching the maximum item number 10. The hypothesis which stated that $p = p_0 = 0.05$ at the power of $1 - \beta = 0.80$, was accepted that is the AQL at $p_0 = 0.05$ can be accepted including non-inspected items and the item inspection can be terminated before inspecting the whole items.

The MSPRT limits for the second category $d_1 = -h_1 + Sn = -1.1486 + 0.1102\text{n}, i = 1 \ldots n$

$d_2 = h_2 + Sn$ =1.8183+0.1102n

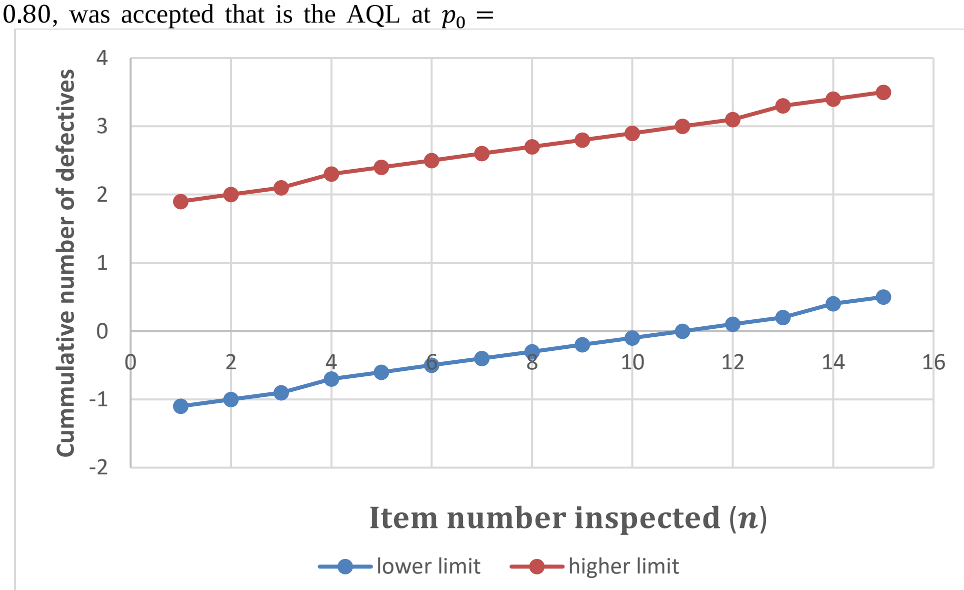

Figure 2: MSPRT computation at power $P^* = 1 - \beta = 0.85$, Lower prop. (AQL) = $p_0 = 0.05$, Higher prop.(LTPD) = $p_1 = 0.20$, max $n = 15$.

Figure 2 is the diagrammatic presentation the result of MSPRT with the upper limit d_2 and the lower limit d_1. The computed average sample number ASN at $p = 0$ was 15 which in comparison was equals to the assigned item number 11 that is $n \geq \frac{h_1}{s}$. This shows that the lot was accepted before reaching the maximum item number 15. The hypothesis which stated that $p = p_0 = 0.05$ at the power of $1 - \beta = 0.85$, was accepted that is the AQL at $p_0 = 0.05$ can be accepted including non-inspected items and the item inspection can be terminated before inspecting the whole items.

Table 4: Decision summary

	ASN	1	2	3
p		$n = 10$	$n = 15$	$n = 25$
0	$\frac{h_1}{s}$	$(9.0349) \approx 9$	$10.56) \approx 11$	$(18.353) \approx 18$
1	$\frac{h_2}{1-s}$	2	2	2
	Decision	$n \geq \frac{h_1}{s}$	$n \geq \frac{h_1}{s}$	$n \geq \frac{h_1}{s}$
The	Conclusion	Accept	Accept	Accept

Table 4 shows the results for the three tested hypothesis using the proposed modified sequential probability ratio test MSPRT, it shows that all the existing values; 10, 15, and 25 were greater than the proposed ASN; 9, 11 and 18 respectively which shows that at a specific ASN, we can accept and adopt the proposed plan for it reduce the inspection time and equally save cost.

CONCLUSION

The proposed MSPRT for lots under Log Normal and Inverse Exponential Rayleigh Distributions plan revealed that the number of sample to be considered for inspection is less than the classical SPRT. Whereas, the existing plan yielded the average number of inspection at a specified level of significance as 10 when the probability of acceptance is 0 defective items, the proposed plan yields 9. Likewise, comparing the classical plan at level 3 yielded 25 as the minimum sample, the proposed plan yields 18 as the minimum as shown in Table 4. Finally, on comparing the two tests, one could easily find that the proposed modified sequential probability ratio test based on Log Normal and Inverse Exponential Rayleigh distributions is relatively the best option than the classical SPRT plan. Hence the proposed MSPRT offers a great balance of cost effectiveness and convenience, making it a practical choice for various applications.

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