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Time Series Modelling and Forecasting Foreign Direct Investment using Linear and Nonlinear Models: The Case of Nigeria

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ABSTRACT
This study addresses the crucial need for accurate forecasting of Foreign Direct Investment (FDIT) trends in Nigeria. FDIT plays a pivotal role in the country's economic growth and development efforts, driving industrialization, infrastructure enhancement, and job creation. However, predicting FDIT accurately is essential for policymakers, investors, and researchers to formulate effective strategies and decisions. This study conducts a comparative analysis of four FDIT forecasting models: Simple Exponential Smoothing (SES), Holt Exponential Smoothing (HES), ARIMA, and NNAR in Nigeria, utilizing R version 4.3.2 and forecast and nnetar packages for implementation. The FDIT dataset is partitioned into 80:20 training-test samples, with models applied to each subset for performance evaluation. Leveraging historical FDIT data, the study evaluates the predictive performance of each model over a specified period, employing error measures such as RMSE, MAE, MASE and MAPE to assess accuracy and reliability. Additionally, residual normality tests and visual inspections validate model assumptions. Results indicate varying levels of accuracy and predictive capability among the models, with HES and NNAR models displaying superior performance compared to SES and ARIMA, evidenced by measure error rates. Residual normality tests affirm the suitability of the models for FDIT forecasting. The study contributes empirical evidence to the existing literature, providing valuable insights into forecasting techniques for FDIT in Nigeria. These insights can guide policymakers, investors, and researchers in selecting appropriate models for forecasting FDIT trends and making informed decisions. Future research avenues could explore advanced modelling techniques and additional variables to further enhance the accuracy and reliability of FDIT forecasts within the Nigerian context.

Keywords:
Foreign Direct Investment (FDIT),
Forecasting Models,
SES,
HES,
ARIMA,
NNAR.

INTRODUCTION

Foreign Direct Investment (FDIT) plays a crucial role in the economic development of countries worldwide, serving as a catalyst for growth, employment generation, and technological advancement. In the Nigerian context, FDIT has emerged as a significant driver of economic progress, attracting investments across various sectors including oil and gas, telecommunications, and manufacturing. As Nigeria strives to achieve sustainable development goals and enhance its global competitiveness, accurate forecasting of FDIT inflows becomes imperative for policymakers, investors, and other stakeholders. The utilization of advanced modelling techniques and statistical methodologies has become increasingly prevalent in forecasting FDIT trends. However, the selection of appropriate models and the evaluation of their predictive accuracy remain pivotal challenges in the field of economic forecasting. This study aims to address this gap by conducting a comparative analysis of FDIT forecasting models in Nigeria, focusing on the application of SES, HES, ARIMA, and NNAR models. The choice of these models is justified by their widespread use and proven effectiveness in forecasting economic variables in diverse contexts. SES and HES are traditional time series forecasting methods known for their simplicity and interpretability (Hyndman & Athanasopoulos, 2018). ARIMA models, on the other hand, are widely employed for their ability to capture complex time series patterns and seasonality (Box et al. 2015). Numerous studies have demonstrated that neural networks, even when they contain non-linearity, are useful for modelling financial data. Because of this, the use of ANN in modelling and forecasting has grown in recent years (Widrow et al. 1994; Zhang, 2004; Kamruzzaman & Sarker, 2004; Hajirahimi and Khashei, 2016). Numerous other studies (Najamuddin, & Fatima, 2022; Ghosh, 2017; Mucaj & Sinaj, 2017)) have compared ARIMA with ANN and found that ANNs outperform ARIMA models. Additionally, Artificial Neural Networks (ANN) have gained prominence for their capability to model non-linear relationships and handle large datasets effectively (Bishop, 1995).

Literature on forecasting techniques for FDIT offers valuable insights into the effectiveness of various models. Akpensuen et al. (2019) examined Nigeria's net FDIT trends using the ARIMA methodology, showcasing its adaptability in forecasting economic indicators, including FDIT inflows, with growth potential identified until 2039. Policy implications underscore the necessity of improving the investment climate and implementing investor-friendly policies for sustained FDIT inflows, suggesting further exploration into non-linear models to handle potential volatility in FDIT series. Jere et al. (2017) investigated three forecasting methods for Zambia's annual net FDIT inflows: SES, Holt-Winters Exponential Smoothing, and ARIMA, with ARIMA proving to be the most fitting model. They stress the importance of accurate forecasting for policymaking and strategic planning. Abdelkader & Hamza (2021) compared the ARDL econometric technique with the ANN model for forecasting FDI flows in Algeria. Their findings, based on MSE and RMSE prediction standards, revealed better accuracy with the ANN model compared to ARDL in forecasting FDIT flows. Yusuf (2022) utilized the ARIMA model and explored various AI algorithms to understand the factors influencing individual foreign direct investment decisions. Machine learning models showcased high accuracy, highlighting significant influences such as ROI, security, and investment facilitation services. Roy (2021) emphasized FDIT's crucial role in economic development, especially in countries like India, proposing LSTM deep learning to enhance forecasting accuracy due to globalization complexities, showcasing superiority over linear models like ARIMA and GARCH.

Musora et al. (2022) utilized ARIMA time series and regression models to analyse FDIT inflows in Zimbabwe, demonstrating reliable forecasts with the ARIMA (0, 1, 1) model, though struggles in explaining outliers. Integration of additional economic factors is suggested to enhance policy planning and implementation. Ye (2021) compared ARIMA and Holt-Winters models for predicting FDI in Shanghai, favouring the Holt-Winters model for its prediction accuracy and highlighting significant cointegration between container throughput and FDIT. Idowu (2021) employed the Box-Jenkins ARIMA model approach to forecast Nigeria's FDI inflows, predicting a slow upward trend in the next decade, ranging between 2.80 billion USD and 3.26 billion USD.

Despite the abundance of literature on FDIT forecasting, there exists a research gap regarding the comparative analysis of these specific models in the Nigerian context. This study seeks to contribute to the body of knowledge by providing empirical evidence on the performance and suitability of SES, HES, ARIMA, and NNAR models for forecasting FDIT inflows in Nigeria. The findings of this research will not only aid policymakers and investors in making informed decisions but also contribute to the advancement of FDIT forecasting methodologies in emerging economies.

MATERIALS AND METHODS

Data

The data covers net inflows of foreign direct investment (FDIT) denominated in US dollars into Nigeria between 1970 and 2022. The information was taken from the World Bank's Nigerian data indicators. Figure 1 displays net FDIT (USD) inflows from 1970 to 2022. Based on the series' descriptive statistics, the average FDIT is \$8,841,062,051, as shown in Table 1, with a minimum FDIT of \$-0.738870004 and a maximum of \$2,001,327,571

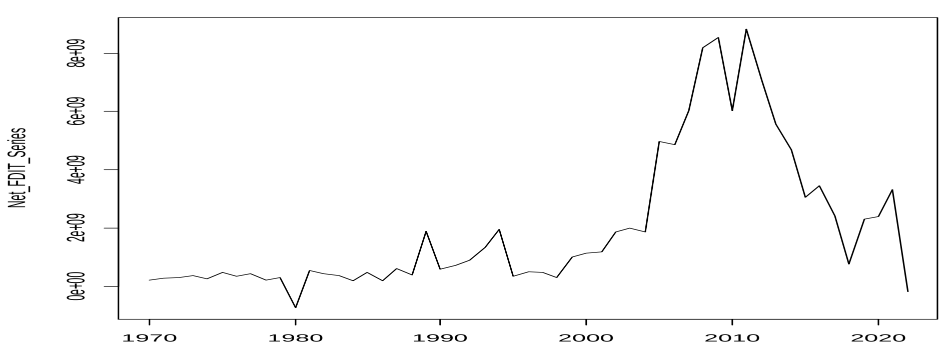


Figure 1: Time Series plot of Nigerian net inflow of FDIT based on USD (Source: R output)

The time series data represents annual net FDIT inflows in USD of Nigeria from 1970 to 2022. It shows fluctuating trends with increasing inflows from 1970 to 1979, a significant decrease in 1980, mixed trends afterward, notable increases in the late 1980s to early 1990s, peaks in the late 1990s and early 2000s, followed by fluctuations with peaks between 2007 and 2019. Notably, there was a negative inflow in 2022. These fluctuations reflect changes in economic conditions, policies, and global factors influencing foreign investment flows.

Table 1: Summary Statistics for Nigerian net inflow of FDIT based on USD

Data	Min	1st Quarter	Median	Mean	3rd Quarter	Max
FDIT (USD)	-0.7388	364434580	775247400	2001327571	2412974916	8841062051

Source: Prepared by the authors based on R program output

Methods

An overview of all the forecasting time series models used in this article is provided. The linear time series is shown first, and then a nonlinear machine learning model.

Linear Time Series Models.

A brief overview of linear time series models, including the SES, HES, and ARIMA, is provided in this section.

The SES Model

The SES model assumes that data will vary around a constant mean value without showing any identifiable trend or steady growth pattern. The SES model forecasts future values based on a weighted average of past observations, with exponentially decreasing weights Here is the formula for the simple exponential smoothing (Hyndman, 2018; Box, 2015):

$$\hat{f}_t+1 = s * a_t + (1 - s) * \hat{f}_t \quad (1)$$

where: $\hat{f}_t+1$ is the forecast for the next time period, a_t is the actual observation at time t, $\hat{f}_t$ is the forecast for time t, and s is the smoothing parameter (0 < s< 1)

The Holt's Exponential Smoothing (HES)

The HES, also known as double exponential smoothing, extends SES by incorporating trend components in addition to seasonal patterns. The mathematical model is represented as:

$$\hat{f}_t+\mathrm{h}|t = l_t + hb_t \quad (2)$$
$$l_t + 1 = s * a_t + (1 - s)(l_t - 1 + b_t - 1) \quad (3)$$
$$b_t = m(l_t - l_t - 1) + (1 - m)b_t - 1 \quad (4)$$

where: $\hat{f}_t+\mathrm{h}|t$ is the forecast at time t for h periods ahead, a_t is the observed value at time t, l_t signifies the level component at time t, b_t denotes the trend component at time t, and s and m are smoothing parameters.

The ARIMA Model

ARIMA models analyse time series data by incorporating autoregressive (AR), differencing (I), and moving average (MA) elements. Mathematical representation:

$$(1\text{-}-\delta_1 K-, \dots \dots \delta_u K^u)(1 - K)^{dp} a_t = (1 + \gamma_1 K + \cdots \gamma_u K_u) r_t \quad (5)$$

where:

where a_t is the observed time series, K is the backshift operator, δ and γ are parameters of autoregressive and moving average components respectively, r_t is the random error, u and v are the orders of AR and MA components, and dp is the differencing parameter (Box et al, 2015).

Non-Linear Time Series Model

The NNAR Model

The ANN Autoregressive (NNAR) model is an ANN in which the input layer consists of a single variable input and successive models up to a lag of p. Hyndman and Athanasopoulos (2018) presented the concept of NNAR. This model, which is represented by the notation NNAR (p, k), is limited to feed forward networks in a single hidden layer. Here, p stands for lag-p as input and k for notes in the hidden layer**.** The NNAR models utilize neural networks to capture nonlinear patterns and dependencies in time series data. These models can learn complex relationships between past observations and future forecasts. The mathematical formulation of NNAR involves training neural networks to predict future values based on historical data (Goodfellow et al., 2016; Brownlee, 2020). The NNAR equation is given by:

$$q_t = c_o + \sum_{j=1}^{N} c_g g(c_{oj} + \sum_{i=1}^{D} c_{ij}\, q_{t-1}) + b_t \quad (6)$$

where:

q_t is the output ($q_t = i, \dots, q_{t-d}$) is the input,

$c_{i.j}$ (i=0,1, 2,...,D=1,2,.......,N) model parameters, which are also refer to as connection weights,

D represents the quantity of input nodes,

N signifies the quantity of hidden nodes

b_t is the random error

The NNAR model assumes a sigmoid function,

$$f(q_t) = \frac{1}{1+e^{-q_t}} \quad (7)$$

$f(q_t)$ is utilized in the linear function transformation of 0 probability to a probability of 1.

The study utilized MAE, RMSE, MASE and MAPE as performance evaluators to assess forecasting error and prediction models. MAPE, based on percentage error, is scale independent, MASE is also scale-free, while MAE indicates that outliers are given less weight (Kumar et al. 2020).

$$\text{MAPE}=\frac{1}{sm}\sum_{i}^{sm}\left(\frac{|a_i-\hat{f}_i|}{|a_i|}\right) \quad (8)$$

where:
sm is out of sample size
a_i represent the actual values of the out of sample data
$\hat{f}_i$ denotes the predicted values of the out of sample data

MAPE calculates the percentage difference between predicted and actual values, making it particularly useful for interpreting errors relative to the magnitude of the observations.

$$\text{RMSE}=\sqrt{\frac{1}{sm}\sum_{i=1}^{sm}\left(a_i-\hat{f}_i\right)^2} \quad (9)$$

$$\text{MASE}=\frac{1}{sm}\sum_{i=1}^{sm}\left(\left|a_t-\hat{f}_t\right|/\frac{1}{sm-1}\sum_{i=1}^{sm}\left|a_t-\hat{f}_t-1\right|\right) \quad (10)$$

where a_t denotes the actual values, $\hat{f}_t$ represent the predicted values and sm indicates the number of observations.

Data Analysis

The R programming language, version 4.3.2, is used to analyse the performance of the recommended framework. Furthermore, we have employed the forecast and nnetar packages to put the models into practice. The FDIT datasets were divided into training and test samples in ratio of 80:20 percent, we applied the models on samples. The auto. arima () function from the forecast package was used to obtain the ARIMA, while ses () and holt () funtions were utilized to obtain SES and HES models respectively. We used the near library in R to vary the number of nodes in the hidden layer. An average of 20 networks, each with 13 weights and a 4-2-1 structure, made up the NNAR model. To observe the models' performances, trained models are applied to the test dataset. The applied models' performances are displayed in Table 2 for net inflows of FDIT based on US dollar**.** Phase 2 of the experiment involved predicting and forecasting the Nigerian FDIT datasets from 2012 to 2030 using developed models. The models were applied to anticipate FDIT trends and fluctuations, crucial for stakeholders' decision-making.

RESULTS AND DISCUSSION

Table 2: The Error Rate of Models on The Net Inflows of FDIT (USD)

Error Measures	Models			
	SES	HES	ARIMA	NNAR
RMSE	9.62+08	9.08+08	9.97+08	**5.21+08**
MAE	6.04+08	5.65+08	6.01+08	**3.97+08**
MAPE	55.48	57.33	59.78	**54.54**
MASE	0.980	0.918	0.976	**0.661**

Source: R program outputs

Tables 2 displays the error rates of the models for forecasting Nigerian FDIT inflows based on US dollar, as measured by Root RMSE, MAE, and MAPE. The results indicate that the NNAR models exhibit lower error rates compared to the other models, suggesting better predictive accuracy.

Table 3: Residuals Normality Test for models on net inflows of FDIT (USD)

Model	Ljung-Box test			
	Test Statistics	p-value	Null Hypothesis	Decision
SES	8.306	0.404	Normality	Accepted
Holt	4.042	0.853	Normality	Accepted
ARIMA (0,1,0)	12.26	0.139	Normality	Accepted
NNAR (4,2)	1.837	0.985	Normality	Accepted

Source: R program outputs

Tables 3 presents the results of the Residuals Normality Test for all the models for net FDIT inflows based on US dollar, indicating the suitability of the residual errors for each model. The Ljung-Box test statistics and p-values suggest that all models have acceptable residual normality, validating the reliability of the models' predictions.

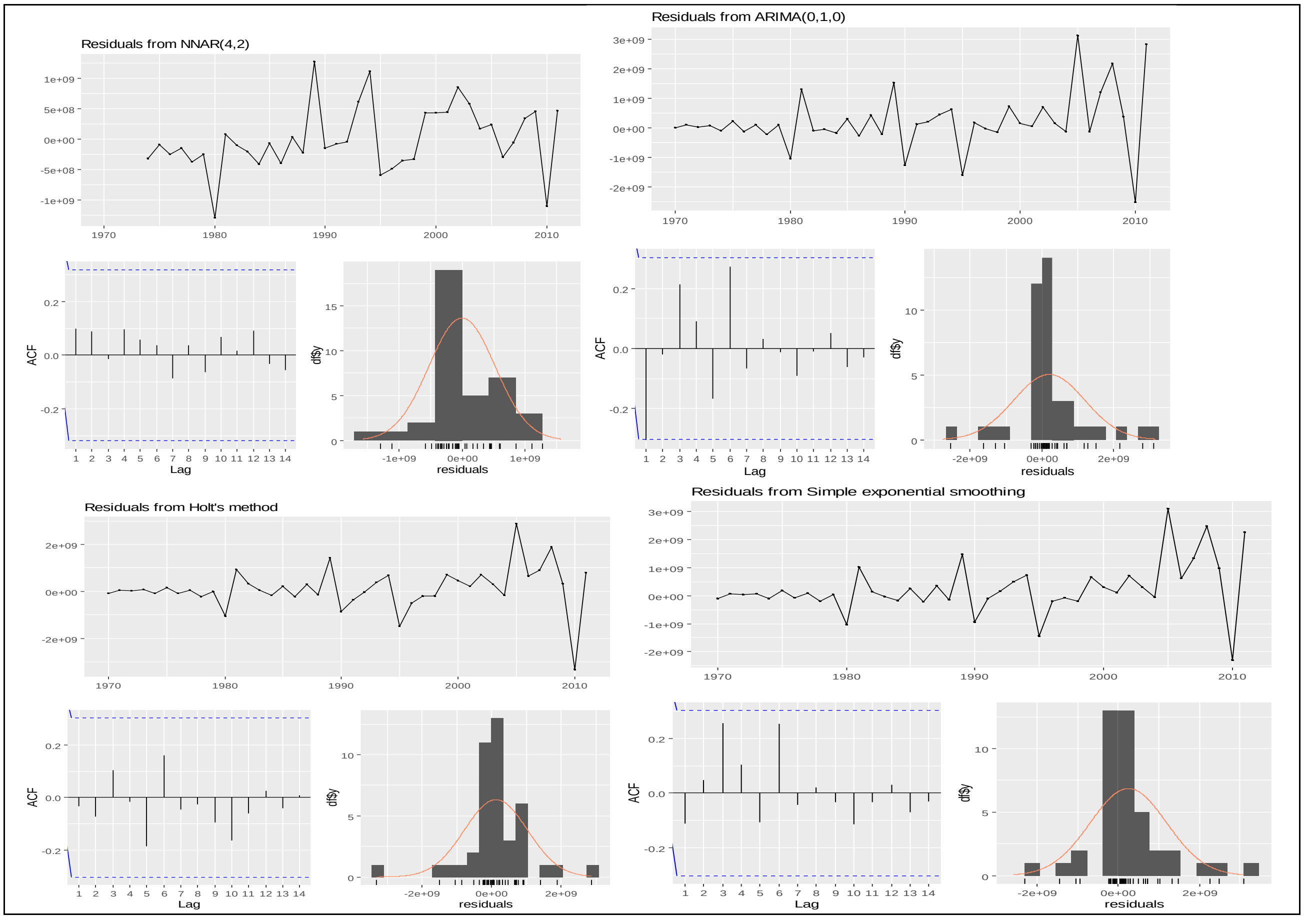
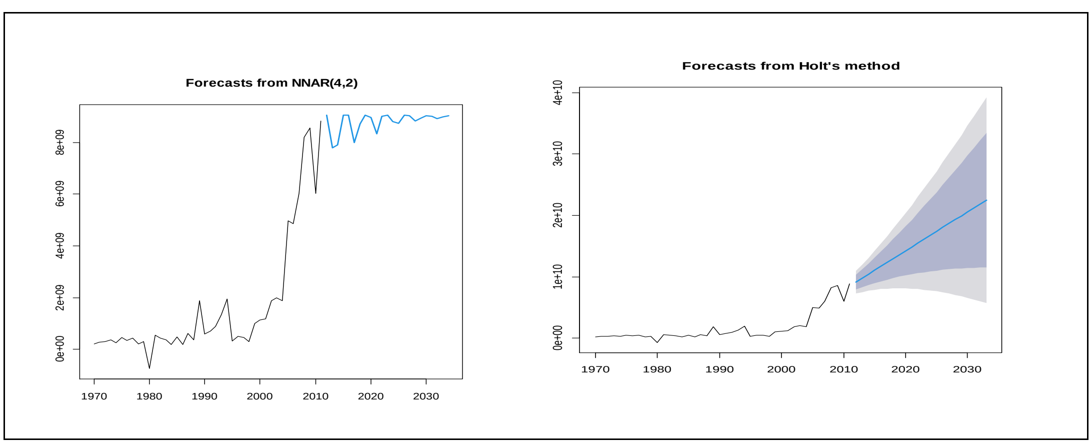


Figure 2: ACF and Histogram Plots of the Applied Models Residuals for net FDIT based on USD (Source: R program outputs)

Applied Models Forecasting Plots

The forecasted values for net FDI inflows from 2023 to 2030 are displayed in Figure 3.

Using the applied models, we plot the forecasted values. As can been seen, the forecasted values follow the trend of FDIT series.



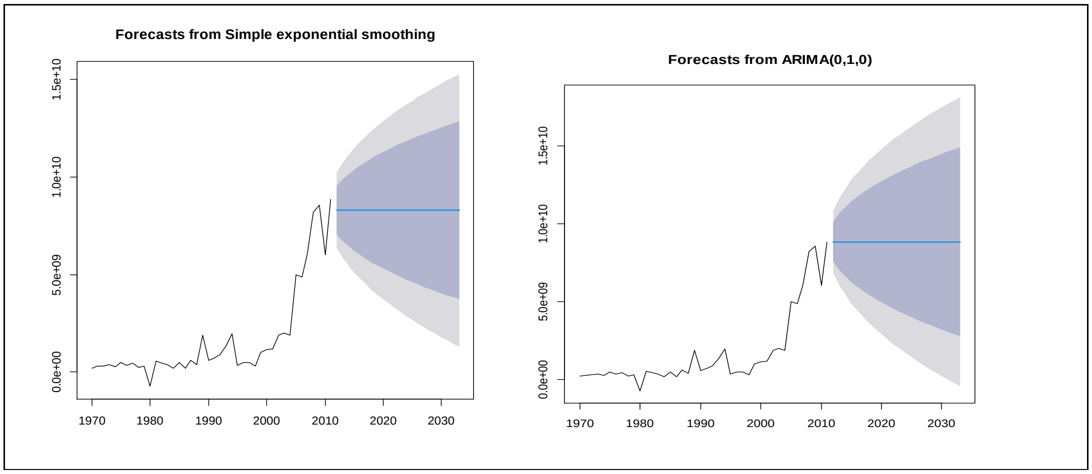


Figure 3: Actual and Forecasted Plots of all the applied Models for net FDIT based on USD(Source: R output)

Discussion of Results

The analysis aimed to forecast net Foreign Direct Investment (FDIT) inflows in US dollars, utilizing SES, HES, ARIMA, and NNAR models. Results showcased NNAR's superior predictive accuracy across all error measures, with notably lower RMSE, MAE, MAPE, and MASE values compared to traditional models, indicating its effectiveness in capturing and predicting underlying data patterns, as shown in Table 2. NNAR's lowest MAPE value suggests its proficiency in minimizing percentage forecasting errors crucial for financial forecasting.
Assessment of residuals' normality via the Ljung-Box test confirmed acceptable residual normality for all models, validating the reliability of their predictions (see Table 3). Acceptance of the null hypothesis implies no significant departures from normality, bolstering confidence in the models' predictive reliability. However, it should be noted that although the residual normality tests offer supporting evidence, a thorough evaluation of the model's reliability should encompass further diagnostic examinations and exploratory data analysis, as illustrated in figure 2. Forecasting plots of Figure 3 visually depicted the forecasted trajectories for 10 years from 2023 to 2033, revealing NNAR as the best-performing model for predicting net FDIT inflows. HES and NNAR emerged as suitable models, with NNAR exhibiting the most accurate predictions based on both performance metric values and visual observations. These findings underscore the importance of advanced modelling techniques, such as neural networks, in financial forecasting, aligning with previous research (Acquah et al.. 2022; Amelot et al., 2021; Mucaj & Sinaj, 2017). They highlight the necessity of employing sophisticated methodologies to enhance prediction accuracy, aiding stakeholders in making well-informed decisions and contributing to the refinement of financial forecasting methodologies. Moreover, the models' enhancements in predicting Nigerian FDIT inflows are consistent with similar studies (Mowafy et al., 2020; Abd El-Aal et al., 2021; Pradhan, 2010) demonstrating the efficacy of these models in improving prediction accuracy and contributing to the evolving landscape of financial decision-making methodologies.

CONCLUSION

This article applies machine learning and statistical models to forecast net Foreign Direct Investment (FDIT) based USD inflows in Nigeria, utilizing four performance metrics for evaluation. Results highlight NNAR and HES as superior models for FDIT forecasting based on a comprehensive analysis of 53 years of FDIT data. The study underscores the significance of accurate FDIT forecasting for governmental policy formulation. In conclusion, the research provides valuable insights into FDIT forecasting in Nigeria, employing various models including SES, ARIMA, HES, and NNAR. The findings stress the importance of meticulous model selection and evaluation in predicting FDIT inflows, aiding policymakers and investors in informed decision-making. Future research could enhance FDI forecasting models by exploring advanced machine learning techniques and incorporating additional variables such as economic and geopolitical factors. Longitudinal studies could provide insights into long-term trends, while regional and sector-specific analyses could offer a comprehensive understanding of investment patterns and opportunities in Nigeria. Addressing these areas of future work will advance FDI forecasting models, contributing to more informed decision-making processes in foreign investment in Nigeria and beyond.

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